

# Calculus III, MiniTest 1 Review Answers

Dr. Graham-Squire, Fall 2012

1. Determine if the following points are collinear, and explain your answer:

$$P = (2, -1, 5) \quad Q = (8, 3, 13) \quad R = (-7, -7, -7).$$

Ans: Yes, because the vectors  $\overrightarrow{PQ}$  and  $\overrightarrow{QR}$  are scalar multiples.

2. An eagle with the head of Nido Qubein is pulling a rope attached to a statue of R Kelly across the quad. The eagle is pulling with a constant force of 100 pounds at an angle of  $60^\circ$  with the ground. If the eagle has to pull the statue 150 feet (so it can sit on a bench next to Gandhi, of course, where it belongs), find the amount of work done to get R Kelly to Gandhi.

Ans: 7500 foot-pounds.

3. When is  $\mathbf{u} \cdot \mathbf{v} = 0$ ? When is  $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ ? Can you use a property or formula involving the cross and/or dot product to explain why that is the case?

Ans:  $\mathbf{u} \cdot \mathbf{v} = 0$  when the two vectors are orthogonal (or when one of them is zero), and  $\mathbf{u} \times \mathbf{v} = \mathbf{0}$  when the two vectors are parallel (or one of them is zero). You can convince yourself of this if you look at the formula for the angle between two vectors  $\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$  and the magnitude of a cross product  $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin(\theta)$ .

4. Use vectors to find the area of the parallelogram that has sides given by the line segment connecting  $(1, 2, -4)$  to  $(2, 1, 3)$  and the line segment connecting  $(1, 2, -4)$  to  $(5, -1, 0)$ .

Ans:  $\sqrt{866}$

5. Sketch the plane given by the equation  $x + 2y + 3z = 6$ .

Ans: Do this on Sage or Grapher to check your work.

6. Find the distance between the parallel lines with parametric equations:

$$x = 3 + 2t \quad y = t \quad z = 4t - 3$$

and

$$x = 3 - 4t \quad y = -1 - 2t \quad z = 2 - 8t$$

Also explain how you know the two lines are parallel.

Ans: The two lines are parallel because if you look at the coefficients of the  $t$  to get your direction vectors, those vectors will be scalar multiples. The distance is  $\sqrt{185/21}$ .

7. Sketch the surface given by the equation  $4x^2 - 9y^2 = -4z^2$ .

Ans: This will be an elliptic cone with  $y$  as the rotational axis. You can either think of it as a quadric surface or a surface of revolution when you graph it.

8. (a) The point  $(2, 2\pi/3, -2)$  is in cylindrical coordinates. Convert it to spherical coordinates.

Ans:  $(2\sqrt{2}, 2\pi/3, 3\pi/4)$

(b) Find an equation in rectangular coordinates for the equation  $z = r^2 \cos^2 \theta$  given in cylindrical coordinates. Sketch and/or describe the graph.

Ans:  $z = x^2$ . The graph will be a cylindrical surface with a parabola for a generating curve and rulings parallel to the  $y$ -axis.