Quiz 6, Calculus III

Fall 2012

12:44
12:48
4 min.

Name: Key

Evaluate the following line integrals. You can use any method you choose (direct, fundamental theorem of line integrals, Green's theorem, etc) as long as that method is applicable.

1. (3 points) $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$ where C is the straight line from (0,0,0) to (1,3,2).

$F = \langle 2xyz,\ x^2z,\ x^2y \rangle$

$\Rightarrow F = \nabla f$ for $f = x^2yz$ ✓ (is conservative)

$$\Rightarrow \int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz = x^2yz \Big|_{(0,0,0)}^{(1,3,2)} \quad \checkmark \ \checkmark$$

$$= 1^2 \cdot 3 \cdot 2 - 0$$

$$= \boxed{6}$$

→ Not conservative.

2. (4 points) $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = xy\mathbf{i} + 2xy\mathbf{j}$ and C is given by $\mathbf{r}(t) = t^2\mathbf{i} + t^2\mathbf{j}$, $0 \le t \le 2$.

$r'(t) = \langle 2t, 2t \rangle dt$, $F = \langle t^2(t^2), 2(t^2)(t^2) \rangle$

$$\Rightarrow \int_C F \cdot dr = \int_0^2 (2t^5 + 4t^5)\,dt = \int_0^2 6t^5\,dt$$

$$= t^6 \Big|_0^2$$

$$= 2^6$$

$$= \boxed{64}$$

3. (3 points) $\int_C 2xy\,dx + (x^2+y^2)\,dy$ where C is the boundary of the square with vertices (0,0), (2,0), (2,2), and (0,2).

$F = \langle 2xy, x^2+y^2 \rangle = \nabla f$ for $f = x^2y + \frac{1}{3}y^3$

$\int 2xy\,dx = x^2y + f(y)$

$\int (x^2+y^2)\,dy = x^2y + \frac{1}{3}y^3 + f(x)$

$\Rightarrow = x^2y + \frac{1}{3}y^3 \Big|_{(0,0)}^{(2,0)} = \boxed{0}$

Closed curve $\Rightarrow$ can use

Green's thm $\Rightarrow \int_C \underbrace{2xy}_{M}\,dx + \underbrace{(x^2+y^2)}_{N}\,dy = \iint_R 2x - 2x\,dA$

$= \iint_R 0\,dA$

$= \boxed{0}$

$\frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2x$