Quiz 3, Calculus III

Fall 2012

4:22
4:30
8 min.

Name: Key

1. (4 points) Dominic has decided to be a mountain climber and he is climbing a hill given by the equation $z = 500 - \left(\frac{x}{4} - 10\right)^2 - \left(\frac{y}{3} - 20\right)^2$. He is standing at the point with (x, y) coordinates of (0,0). Answer the following:

   (a) If Dominic wants to walk in the steepest direction, which direction should he go? Write your answer as vector with 2 entries.

   (b) Baby Eva is at the point on the mountain with (x, y) coordinates of (20,15). She is holding Dominic's "Mo math, mo problems" shirt and threatening to drool on it. What will the slope be if Dominic walks straight toward Eva?

(a) $z_x = -2\left(\frac{x}{4} - 10\right) \cdot \frac{1}{4} = \frac{-1}{2}\left(\frac{x}{4} - 10\right)$

$z_y = -2\left(\frac{y}{3} - 20\right) \cdot \frac{1}{3} = \frac{-2}{3}\left(\frac{y}{3} - 20\right)$

$z_x(0,0) = 5$

$z_y(0,0) = \frac{40}{3}$ $\Rightarrow$ $\nabla f(0,0) = \langle 5, \frac{40}{3} \rangle$ is the direction of steepest ascent.

(b) Want directional derivative in direction of $\langle 20, 15 \rangle = \vec{v}$

$\|v\| = 25$ $\Rightarrow$ unit vector in Eva direction is $\vec{u} = \langle \frac{4}{5}, \frac{3}{5} \rangle$

$D_{\vec{u}} f(x,y) = \langle \frac{4}{5}, \frac{3}{5} \rangle \cdot \langle 5, \frac{40}{3} \rangle$

$= 4 + 8 = \boxed{12}$

2. (3 points) Calculate $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$.

Change to polar coords $r^2 = x^2+y^2$ to get

$$\lim_{r\to 0^+} \frac{\sin(r^2)}{r^2} \overset{H}{=} \lim_{r\to 0^+} \frac{\cancel{2r}(\cos(r^2))}{\cancel{2r}}$$

$$= \cos 0$$

$$= \underline{1}$$

3. (3 points) Calculate $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ if $w = \ln(x^2+y)$, $x = 2ts$, and $y = 4-t$. Simplify your answer if possible.

$$\frac{\partial w}{\partial x} = \frac{2x}{x^2+y} \qquad \frac{\partial w}{\partial y} = \frac{1}{x^2+y}$$

$$\frac{\partial x}{\partial s} = 2t, \quad \frac{\partial x}{\partial t} = 2s \quad , \frac{\partial y}{\partial t} = -1 \quad , \frac{\partial y}{\partial s} = 0$$

$$\frac{\partial w}{\partial t} = \left(\frac{2x}{x^2+y}\right)(2s) + \left(\frac{1}{x^2+y}\right)(-1) = \frac{4(2ts)s+1}{(2ts)^2+4-t} = \boxed{\frac{8ts^2+1}{4t^2s^2+4-t}}$$

$$\frac{\partial w}{\partial s} = \left(\frac{2x}{x^2+y}\right)(2t) + \left(\frac{1}{x^2+y}\right)\cdot 0 = \frac{4(2ts)t}{(2ts)^2+4-t} = \boxed{\frac{8t^2s}{4t^2s^2+4-t}}$$