Quiz 2, Calculus III

Fall 2012

3:33
3:40
7 min

Name: Key

1. (4 points) Given the position function $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, \sqrt{5}t \rangle$, find the unit tangent vector $\mathbf{T}(t)$. (Hint: if you have forgotten the formula for $\mathbf{T}$, it is simply the velocity vector normalized.)

$$\vec{r}\,'(t) = \langle 2\cos t, -2\sin t, \sqrt{5} \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{4\cos^2 t + 4\sin^2 t + 5}$$

$$= \sqrt{4(\cos^2 t + \sin^2 t) + 5}$$

$$= \sqrt{9}$$

$$= 3$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \boxed{\frac{1}{3}\langle 2\cos t, -2\sin t, \sqrt{5} \rangle}$$

2. (4 points) Use the acceleration function $\mathbf{a}(t) = e^t\mathbf{i} - 8\mathbf{k}$ and the initial conditions $\mathbf{v}(0) = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{0}$ to find the velocity and position functions.

$\vec{v}(t) = \int \vec{a}(t) = \int (e^t \vec{i} + 0\vec{j} - 8\vec{k})\,dt = e^t\vec{i} + 0\vec{j} - 8t\vec{k} + \vec{C}_1$

$\langle 2,3,1\rangle = \vec{v}(0) = \langle 1,0,0\rangle + \vec{C}_1 \quad \Rightarrow \quad \vec{C}_1 = \langle 1,3,1\rangle$

$\boxed{\vec{v}(t) = (e^t+1)\vec{i} + 3\vec{j} + (1-8t)\vec{k}}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle e^t + t, 3t, t - 4t^2\rangle + \vec{C}_2$

$\vec{r}(0) = \langle 1,0,0\rangle + \vec{C}_2 = \langle 0,0,0\rangle$

$\Rightarrow \vec{C}_2 = \langle -1,0,0\rangle$

so $\boxed{\vec{r}(t) = \langle e^t + t - 1, 3t, t - 4t^2\rangle}$

3. (2 points) Below is a sketch of the position function $\mathbf{r}(t)$ from $t = 0$ to $t = 10$ in the plane. The dot in the middle of the curve represents the position at $t = 5$. Sketch in the velocity vector $\mathbf{v}(5)$ and the acceleration vector $\mathbf{a}(5)$ with their initial point located at the dot given for $\mathbf{r}(5)$.