Test 3 - MTH 1420

Dr. Adam Graham-Squire, Spring 2020

Name: ______________________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Read every question carefully, follow all instructions, and answer every question. Clearly indicate your answer by putting a box around it.
4. Calculators and/or Maple are allowed on ALL questions of the test, however you should still show all of your work. Specifically, you should explain steps you do to set up integrals and give reasons for integral tests–you should consider Maple as a means to *check* your answer. Most of the points will be for the work, not the answer (since that is easy to find if you have Maple).
5. You should not have any other websites or programs up on your computer screen other than blackboard/Honorlock and Maple.
6. You should not have anything in your work area other than blank paper, a calculator (if you want it) and (if you print) a copy of the test.
7. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
8. Make sure you sign the pledge.
9. Number of questions = ???. Total Points = ???.

~~5~~ 2

1. (~~5~~ points) Determine whether the sequence (SEQUENCE, not *series*) converges or diverges. If it converges, find the limit. For full credit, you must explain/show your work. You can use Maple to check your answer, but you will only receive credit for work shown.

(a) $a_n = \dfrac{\cos(5n)}{2+\sqrt[3]{n}}$

(b) $a_n = \sqrt{\dfrac{7+3n^2}{12n^2+5n}}$

(a) $-1 \le \cos(5n) \le 1$

$\Rightarrow \dfrac{-1}{2+\sqrt[3]{n}} \le \dfrac{\cos(5n)}{2+\sqrt[3]{n}} \le \dfrac{1}{2+\sqrt[3]{n}}$

$\Rightarrow \lim_{n\to\infty}(\ ) \le \lim_{n\to\infty}(\ ) \le \lim_{n\to\infty}(\ )$

$\dfrac{-1}{\infty} \le \lim_{n\to\infty} \dfrac{\cos(5n)}{2+\sqrt[3]{n}} \le \dfrac{1}{\infty}$

$0 \le$ " ≤ 0

$\Rightarrow \lim_{n\to\infty} \dfrac{\cos(5n)}{2+\sqrt[3]{n}} = 0$

by Squeeze.

$\boxed{\text{converges}}$

(b) $\lim_{n\to\infty} \sqrt{\dfrac{(7+3n^2)\frac{1}{n^2}}{(12n^2+5n)\frac{1}{n^2}}}$

$= \lim_{n\to\infty} \sqrt{\dfrac{\left(\frac{7}{n^2}+3\right)}{\left(12+\frac{5}{n}\right)}}$

$= \sqrt{\dfrac{3}{12}}$

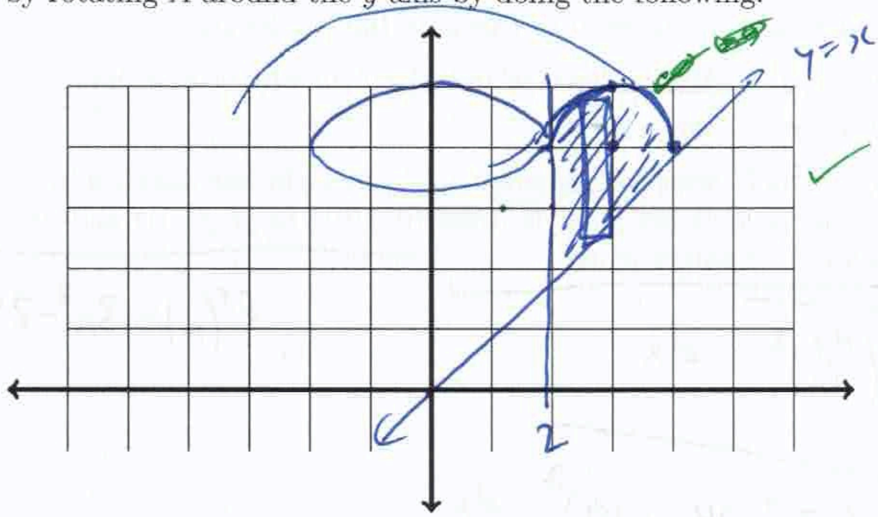
$= \sqrt{\dfrac{1}{4}}$

$= \boxed{\dfrac{1}{2}}$

Converges to

+1 if some work and say diverges

2. (5 points) Let A be the region between the functions $y = 5-(x-3)^4$, $y = x$ from $x = 2$ to $x = 4$ (you may want to graph that). Calculate the volume of the solid generated by rotating A around the y-axis by doing the following:



(a) Set up, but do not integrate, the integral to calculate this volume.

(b) Is your integral in (a) integrable by hand? Explain why or why not.

(c) Use Maple to calculate the integral from (a). (round to nearest 0.01)

top − bottom

(a) Shells: $2\pi r h$ $\quad r = x \quad h = (5-(x-3)^4 - x)$

$$V = 2\pi \int_2^4 x(5-(x-3)^4 - x)\,dx$$

(b) Yes! If you multiply it all out, it is just a polynomial, and you use power rule on each term.

(c) $\dfrac{304\pi}{15} = 63.67$

3. (~~5~~ 3 points) Calculate the arc length of $f(x) = x^3 - 12x^2 + 39x$ from $x = 0$ to $x = 7$. Specifically, do the following:

(a) Set up, but do not integrate, the integral to calculate this arc length.

(b) Is your answer from (a) integrable by hand or not? Explain why or why not.

(c) Use Maple to integrate your answer in (a).

(d) Does your answer in (c) make sense? Compare it to the straight line distance from the start and end point of your arc (that is, from $(0, f(0))$ to $(7, f(7))$) and use that to help you judge if (c) makes sense.

Do NOT integrate on Maple!

(a) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

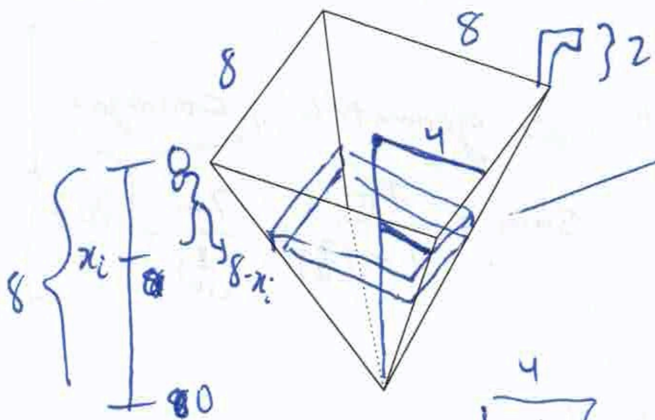
$f'(x) = 3x^2 - 24x + 39$

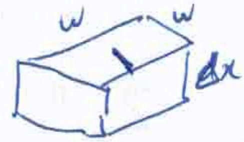
$= \int_0^7 \sqrt{1 + (3x^2 - 24x + 39)^2}\, dx$

(b) Not integrable by hand b/c no x term out side to do substitution with.

~~(c)~~

4. (5 points) A swimming pool is in the shape of an inverted pyramid with a square base (see below for diagram), where each side of the base is 8 feet long, and the pool is 8 feet deep at its deepest point. The pool is filled up with water. How much work will it take to pump out all the water in the pool to a point 2 feet *above* the top of the pool? Note that the density of water is 62.5 lbs/ft^3. Round to nearest ~~lb~~. Whole #

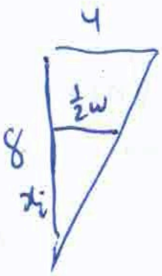




$V = w^2 \Delta x$

$= x_i^2 \Delta x$

$F = 62.5\, x_i^2 \Delta x$

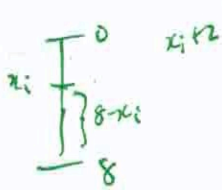


$\frac{4}{8} = \frac{\frac{1}{2}w}{x_i} \Rightarrow \frac{x_i}{2} = \frac{1}{2}w \Rightarrow x_i = w$

$\text{Work} = (10 - x_i) \cdot 62.5 \cdot x_i^2 \Delta x$

$62.5 \int_0^8 (10 - x) \cdot x^2 \, dx$

$= 42{,}667$ (used Maple)



$62.5 \int_0^8 (10x^2 - x^3)\, dx$

$= 62.5 \left(\frac{10}{3}x^3 - \frac{x^4}{4} \right) \Big|_0^8$

$= 62.5 \left(\frac{10}{3}(8^3) - \frac{8^4}{4} \right) - 0 =$

5. (**6** points) State if the series is convergent or divergent. For full credit, you must explain what test(s) you use and show your work. You can use Maple to check your answer, but you will only receive credit for work shown.

- If divergent, explain why.
- If convergent, either (i) calculate the sum exactly or (ii) explain why the exact sum cannot be calculated.

$\Rightarrow$ 3 (a) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi \cdot (7^{n+1})}{10^{n}}$ $= \displaystyle\sum_{n=0} \pi \cdot 7 \left(\frac{7}{10}\right)^n \Rightarrow$ geometric, $\boxed{\text{Convergent,}}$

Sum $= \dfrac{7\pi}{1-\left(\frac{7}{10}\right)} = \dfrac{7\pi}{\left(\frac{3}{10}\right)} = \boxed{\dfrac{70\pi}{3}}$

3 (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n! \cdot (100^{n+2}) \cdot \sqrt{n}}{(3^n) \cdot (n+1)!}$

(b) Ratio Test: $\left|\dfrac{(\cancel{n+1})! \, (100^{n+3}) \cdot \sqrt{n+1}}{3^{n+1} \cdot (n+2)!} \cdot \dfrac{(3^n) \cdot (n+1)!}{\cancel{n!} (100^{n+2}) \cdot \sqrt{n}}\right|$

$$= \lim_{n\to\infty} \frac{100}{3} \frac{(n+1)}{(n+2)} \cdot \sqrt{\frac{n+1}{n}}$$

$$= \frac{100}{3} (1) \cdot \sqrt{1}$$

$$= \frac{100}{3} > 1 \Rightarrow \boxed{\text{divergent}}$$

15 min

6. (6 points). State if the series is convergent or divergent. For full credit, you must explain what test(s) you use and show your work. You can use Maple to check your answer, but you will only receive credit for work shown.

- If divergent, explain why.
- If convergent, explain whether or not it is *absolutely* convergent or *conditionally* convergent.

consider $a_n =$

(a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^2}{2n^2-1}\right)$ $\longrightarrow$ $\cancel{\Sigma} \ln\left(\frac{n^2}{2n^2-1}\right) = \ln\left(\frac{n^2 \cdot \frac{1}{n^2}}{(2n^2-1)\frac{1}{n^2}}\right)$

$= \ln\left(\frac{1}{2-\frac{1}{n^2}}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\pi}}$

as $n \to \infty$, this approaches $\ln\left(\frac{1}{2}\right) \neq 0$

so (a) $\boxed{\text{diveges}}$ by the Test for divergence.

(b) Consider $\sum \left| \frac{(-1)^{n+1}}{n^{\pi}} \right|$

$= \sum \frac{1}{n^{\pi}}$

$= \cancel{\quad}$

Converges by p-series test b/c $\pi > 1$

So it is absolutely Convegent.

7. ($\cancel{5}$ 6 points). State if the series is convergent or divergent. For full credit, you must explain what test(s) you use and show your work. You can use Maple to check your answer, but you will only receive credit for work shown.

- If divergent, explain why.
- If convergent, either (i) calculate the sum exactly or
  (ii) approximate the sum to the nearest ~~0.01~~ .01 if you cannot calculate it exactly.

will need to use Maple.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ $\Rightarrow$ Alt. Series Test: as $n \to \infty$ $n+1$ increases

$\Rightarrow \frac{1}{\sqrt{n+1}}$ is decreasing

b/c denom. gets bigger

and $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = \frac{1}{\infty} = 0$

so (a) converges by A.S.T.

Need to find when

$\frac{1}{\sqrt{n+1}} \le .01$

$\Rightarrow \frac{1}{0.1} \le \sqrt{n+1}$

$100 \le \sqrt{n+1}$

$100 \le n+1$

$99 \le n$

Use Maple to find

$\sum_{i=1}^{99} \frac{(-1)^n}{\sqrt{n+1}} = -0.445$

(b) $\sum_{n=2}^{\infty} \frac{n^2+1}{n^3-1}$ $\Rightarrow$ "like" $\frac{1}{n}$

Limit Comparison Test

$$\lim_{n\to\infty} \left| \frac{\frac{n^2+1}{n^3-1}}{\left(\frac{1}{n}\right)} \right|$$

$$= \lim_{n\to\infty} \frac{n^3+n}{n^3-1}$$

$$= \lim_{n\to\infty} \frac{(n^3+n)\frac{1}{n^3}}{(n^3-1)\frac{1}{n^3}}$$

$$= \lim_{n\to\infty} \frac{1+\frac{1}{n^2}}{1-\frac{1}{n^3}}$$

$= 1$, so L.C.T says they do the same thing

$\Rightarrow \sum \frac{n^2+1}{n^3-1}$ diverges

b/c $\sum \frac{1}{n}$ diverges (Harmonic Series)

Extra Credit(2 points) Is the series convergent or divergent? For full credit, you must explain what test(s) you use and show your work. You can use Maple to check your answer, but you will only receive credit for work shown.

$$\sum_{n=1}^{\infty} \frac{\ln(n^3)}{n}$$

Integral Test: compare to

$$\int_1^\infty \frac{\ln(x^3)}{x}\,dx$$

$$= \lim_{n\to\infty} \int_1^n \frac{3\ln(x)}{x}\,dx$$

$u = \ln x$
$du = \frac{1}{x}\,dx$
$x\,du = dx$

$u = \ln n$
$u = \ln 1 = 0$

$$= \lim_{n\to\infty} \int_0^{\ln n} \frac{3u}{\cancel{x}} \cdot \cancel{x}\,du$$

$$= \lim_{n\to\infty} \int_0^{\ln n} 3u\,du$$

$$= \lim_{n\to\infty} \frac{3}{2}u^2 \Big|_0^{\ln(n)} = \lim_{n\to\infty} \frac{3}{2}(\ln(n))^2 - 0$$

$= \infty$ b/c $\ln n \to \infty$ as $n \to \infty$

diverges