Test 2 - MTH 1420

Dr. Adam Graham-Squire, Spring 2020

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Calculators and/or Maple <u>are</u> allowed on the last 3 questions of the test, however you should still show all of your work. No calculators (other than a basic 4-function) are allowed on the first 6 questions of the test. You should finish the No Calculator/Maple portion first, and when you are done with it, turn it in to me and then you can open up your computer/calculator to finish the last 3 questions, if you need it.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 7. Total Points = 37.

No Calculator or Computer Allowed

1. (5 points) Calculate the following integral:

$$\int_0^\infty \frac{4x}{(x^2+1)^2}\,dx$$

$$= \lim_{n\to\infty} \int_0^n \frac{4x}{(x^2+1)^2}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$u = n^2+1 = 1+n^2$

$u = 0^2+1 = 1$

$$= \lim_{n\to\infty} \int_1^{1+n^2} \frac{\cancel{4}^{2}\cancel{x}}{u^2} \cdot \frac{du}{\cancel{2x}}$$

$$= \lim_{n\to\infty} 2\int_1^{1+n^2} u^{-2}\,du$$

$$= \lim_{n\to\infty} 2 \quad -u^{-1}\Big|_1^{1+n^2}$$

$$= 2\left(\lim_{n\to\infty}\left(-\frac{1}{1+n^2}\right)^{\to 0} - (-1)\right)$$

$$= \boxed{2}$$

2. (5 points) Calculate the integral:

$$\int \frac{\tan(x)}{\cos^3(x)}\,dx$$

$$= \int \frac{\left(\frac{\sin x}{\cos x}\right)}{\cos^3 x}\,dx$$

$$= \int \frac{\sin x}{\cos^4 x}\,dx$$

$u = \cos x$

$du = -\sin x \; dx$

$\frac{du}{-\sin x} = dx$

$$= \int \frac{\cancel{\sin x}}{u^4} \cdot \frac{du}{\cancel{-\sin x}}$$

$$= -\int u^{-4}\,du$$

$$= -\left(\frac{u^{-3}}{-3}\right) + C$$

$$= \boxed{\frac{1}{3\cos^3 x} + C}$$

3. (5 points) Prove the derivative rule that $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)$.

Hint: Start with $\int \frac{1}{\sqrt{1-x^2)}}\,dx$ and do trigonometric substitution. Note that since your integral is given in terms of x, you *answer* should also be in terms of x.

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$x = 1 \cdot \sin\theta$

$dx = \cos\theta\, d\theta$

$$= \int \frac{1}{\sqrt{1-\sin^2\theta}} \cdot \cos\theta\, d\theta$$

$$= \int \frac{1}{\cancel{\sqrt{\cos^2\theta}}} \cdot \cancel{\cos}\theta\, d\theta$$

$x = \sin\theta$

$\Rightarrow \arcsin(x) = \theta$

$$= \int 1\, d\theta$$

$$= \theta$$

(Now rewrite it ~~terms~~ of x: $\theta = \sin^{-1}(x)$)

$$= \boxed{\sin^{-1}(x) + C}$$

Name: Key

4. (6 points) Consider the integral $\int_0^2 e^{\sin^2(x)}\,dx$.

Briefly

(a) Explain why the integral is *not* integrable by hand (or at least not *easily* integrable). ~~What might~~ you do if you were *trying* to integrate it by hand, and why would those tricks not work?

(b) Use Maple to approximate the integral by doing a Riemann sum with 15 subintervals, using the midpoint rule (that is, calculate M_{15}). round to five decimal places

(c) Use Maple and the error estimate formula to calculate the maximum error for the Riemann sum above. You should do calculations on Maple, but explain what you are doing and your results here.

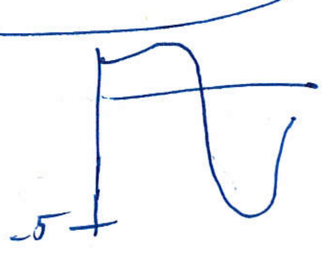
(d) Use Maple to get a decimal approximation for the value of $\int_0^2 e^{\sin^2(x)}\,dx$. Is it within the error you found in part (c)?

(a) if you try to do u-subs, you would have $u = \sin^2 x$, ($du = 2\sin x \cos x\, dx$), and there are no extra $\sin x$, $\cos x$ terms lying around to make that would. Could ~~write with~~ use ½-angle formula but would still have issues... Int. by parts just makes it uglier.

(b) Did Middlesum ($e^{\cdots}$, $x=0..2$, 15), got 3.85663

(c) $b-a = 2-0$, $n=15$, $f''(x) = e^{\sin^2 x}(2\cos^2 x - 2\sin^2 x + 4\cos^2 x \sin^2 x)$

Graph, looks like



low less than −5

Choose $\boxed{K=6} > |-5|$

Then $E_M = \dfrac{6(2^3)}{24(15^2)} = \dfrac{2}{225} = 0.00\bar{8}$

(d) 3.85534, is 0.001 off from estimate, less than.

→ specifically, set it up then use Maple

5. (6 points) You can use Maple to help with this problem, but you should show/explain what you are doing here on the paper as well.

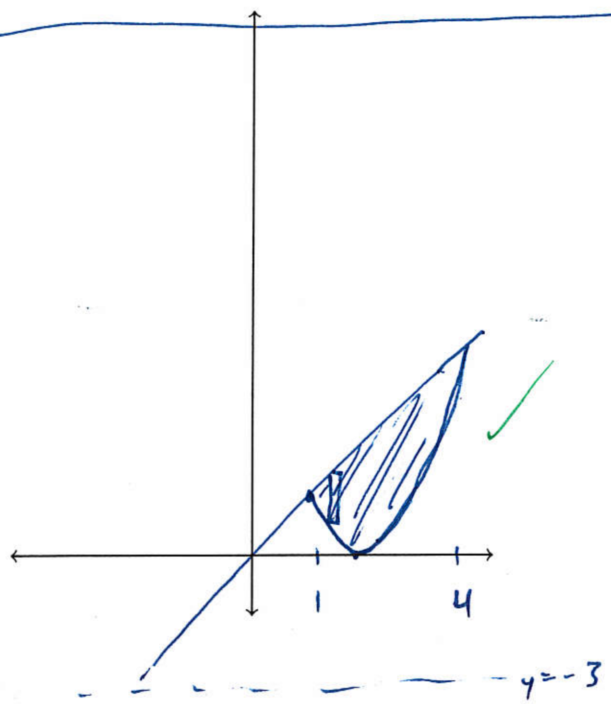
(a) Find the volume of the solid formed by rotating A about <u>the x-axis</u>, where A is given by the region between the graphs of $f(x) = (x-2)^2$ and $g(x) = x$.

(b) Find the volume of the solid formed by rotating A about the line $y = -3$, where A is given by the region between the graphs of $f(x) = (x-2)^2$ and $g(x) = x$.

(c) Suppose I asked you to find the volume of the solid formed by rotating A about the line $y = -n$, where A is given by the region between the graphs of $f(x) = (x-2)^2$ and $g(x) = x$. What happens to that volume as $n \to \infty$? Explain this in two ways:

(i) by describing what is happening to the integral and

(ii) by describing what the picture/diagram would look like.



$(x-2)^2 = x$

$x^2 - 4x + 4 - x = 0$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

(a) $\pi \int_1^4 \left(x^2 - ((x-2)^2)^2\right) dx$ (put in Maple, get: $\frac{72\pi}{5} \approx 45.23$ 0.5

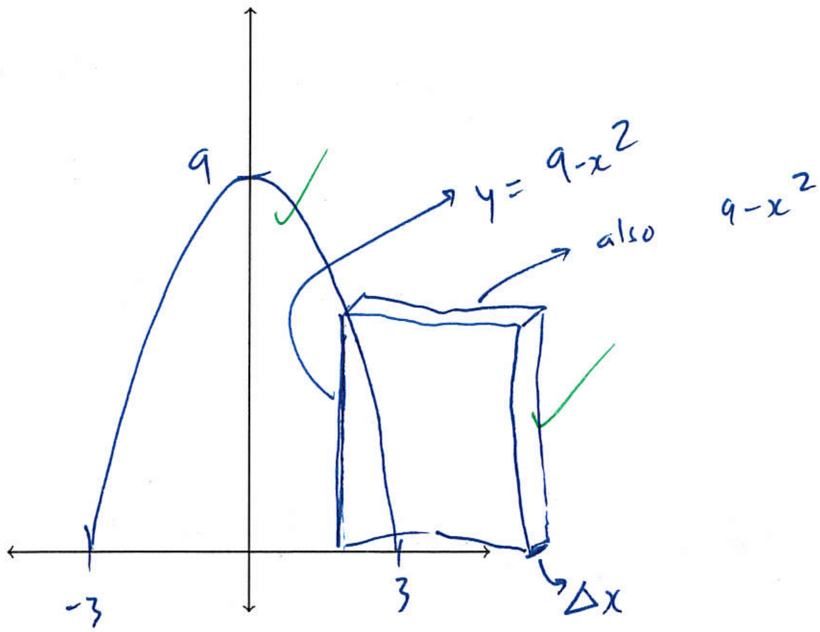
(b) $\pi \int_1^4 \left((3+x)^2 - (3+(x-2)^2)^2\right) dx =$: $\frac{207\pi}{5} \approx 130.06$ 0.5

(c)(i) As $n \to \infty$, this integral will be the same but with n instead of 3. As $n \to \infty$, the value of the integral will get bigger (goes to ∞) b/c it will have bigger #s.

(ii) The diagram would look like a ring with a larger and larger diameter. The thickness of the ring stays the same but you get more of it b/c the diameter keeps growing,

6. (5 points) You can use Maple to help with this problem, but you should show/explain what you are doing here on the paper as well.Find the volume of the solid with the following description:

The base of the solid is the region A in quadrants I and II, where A is the area bounded by the x-axis and the curve $f(x) = 9 - x^2$. Cross sections of the solid perpendicular to the x-axis are squares with one side lying on A (that is, one corner on f and the other on the x-axis).



$$\Rightarrow \int_{-3}^{3} (9-x^2)^2\, dx = \text{(put into Maple)}$$

$$= \frac{1296}{5} = \boxed{259.2}$$

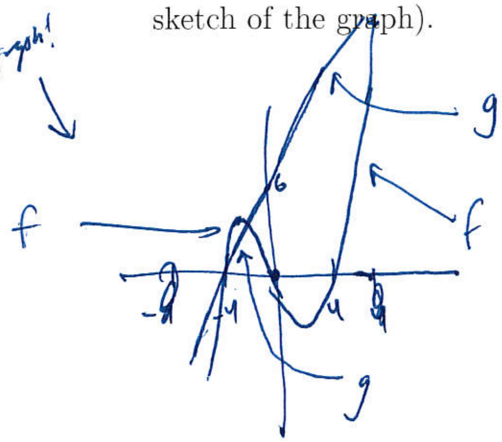
25

7. (5 points) Find the area completely enclosed by the graphs of the functions

$$f(x) = \frac{x^3}{8} - 2x \quad = x\left(\frac{x^2}{8} - 2\right) \Rightarrow = 0 \text{ at } x = -4 \qquad \text{and} \qquad g(x) = 1.5x + 6 \quad = 0 \text{ at } x = -4$$

You can use Maple to help you solve the answer (including calculating the integral, once you have set it up), but you should explain/show your steps/work below (including a sketch of the graph).

graph!



$$\frac{1}{8}\left(x^3 - 16x\right) = \frac{1}{4}(6x + 24)$$

$$\Rightarrow x^3 - 16x - 12x - 24$$

intersect at $x = -4, -2, 6$

0.5

f on top — g on top

$$\int_{-4}^{-2} \left(\frac{x^3}{8} - 2x - (1.5x+6)\right) dx + \int_{-2}^{6} \left(1.5x + 6 - \left(\frac{x^3}{8} - 2x\right)\right) dx$$

(Put into Maple)

$$= \boxed{65.5}$$

0.5

Extra Credit(up to 2 points). You can choose either to get a safe 0.5 points of extra credit, or take a risk and get two points. If you choose 0.5 points, you are guaranteed to get it. If you choose 2 points, and two or fewer students in the class choose 2 points, then you get the 2 points. If three or more people in the class choose two points, though, everyone who chose 2 points gets *no* extra credit points.

2 — ~~IIII~~ I

0.5 — ~~IIII~~ I