Test 1 - MTH 1420

Dr. Adam Graham-Squire, Spring 2020

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

16 min.

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Calculators and/or Maple are allowed on the last 3 questions of the test, however you should still show all of your work. No calculators (other than a basic 4-function) are allowed on the first 6 questions of the test. You should finish the No Calculator/Maple portion first, and when you are done with it, turn it in to me and then you can open up your computer/calculator to finish the last 3 questions, if you need it.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 9. Total Points = 43.

No Calculator or Computer Allowed

1. (5 points) Calculate $\int_3^4 \frac{1}{(3x-7)^2}\,dx$

$u = 3x - 7$

$du = 3\,dx$

$\frac{du}{3} = dx$

$u = 3(3) - 7 = 2$

$u = 3(4) - 7 = 5$

$$= \int_2^5 \frac{1}{u^2} \cdot \frac{du}{3}$$

$$= \frac{1}{3}\int_2^5 u^{-2}\,du$$

$$= \frac{1}{3}\left(-u^{-1}\right)\Big|_2^5$$

$$= \frac{1}{3}\left(-\frac{1}{5} + \frac{1}{2}\right)$$

$$= \frac{1}{3}\left(\frac{5}{10} - \frac{2}{10}\right) = \frac{1}{3} \cdot \frac{3}{10} = \boxed{\frac{1}{10}}$$

-0.5 + for $-\frac{1}{2}$ instead of $+\frac{1}{2}$

-2 if not u-subs (at least)

2. (5 points) Calculate $\int_1^e \frac{(2+3\sqrt{x})}{x\sqrt{x}}\,dx$

$$= \int_1^e \left(\frac{2}{x^{3/2}} + \frac{3\sqrt{x}}{x\sqrt{x}} \right) dx$$

$$= \int_1^e \left[2x^{-3/2} + 3\left(\frac{1}{x}\right) \right] dx$$

$$= 2(-2x^{-1/2}) + 3 \ln|x| \Big|_1^e$$

$$= -4e^{-1/2} + 3\ln e - (-4 \cdot 1^{-1/2} + 3 \ln 1)$$

0

$$= \frac{-4}{\sqrt{e}} + 3 + 4 + 0$$

$$= 7 - \frac{4}{\sqrt{e}}$$

$\int u\,dv = uv - \int v\,du$

3. (5 points) Calculate $\int x^2 e^x \, dx$

$u = x^2 \quad \int dv = \int e^x dx$

$du = 2x\,dx \quad v = e^x$

$= x^2 e^x - \int e^x \cdot 2x\,dx$

$u = x \quad \int dv = \int e^x dx$

$du = dx \quad v = e^x$

$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$

$= \boxed{x^2 e^x - 2xe^x + 2e^x + C}$

4. (5 points) Calculate $\int x \sec^2(x^2)\tan(x^2)\,dx$

$u = \tan(x^2)$

$du = \sec^2(x^2) \cdot 2x\,dx$

$\frac{du}{2x\sec^2(x^2)} = dx$

$= \int \cancel{x}\, \cancel{\sec^2(x^2)} \cdot u \cdot \frac{du}{2\cancel{x}\,\cancel{\sec^2(x^2)}}$

$= \frac{1}{2}\int u \; du$

$= \frac{1}{2} \cdot \frac{1}{2} u^2 + C$

$= \boxed{\frac{1}{4}\left(\tan(x^2)\right)^2 + C}$

or $\frac{\tan^2(x^2)}{4} + C$

2.5

5. (~~5~~ points) Calculate $\int \left(7 + e^x + \frac{1}{1+x^2}\right) dx$

$$= \boxed{7x + e^x + \arctan x + C}$$

2.5

6. (~~3~~ points) Calculate

$$\frac{d}{dx}\left(\int_1^{\cos(x)} \left[e^{(t^2)} + \ln(4+\sqrt{t})\right] dt\right)$$

$$= \left[e^{(\cos x)^2} + \ln\left(4+\sqrt{\cos(x)}\right)\right]\left(-\sin x\right)$$

By FTC, $\frac{d}{dx}\int$ cancels out, but need to use chain rule to get $\frac{d}{dx}(\cos x) = -\sin x$

-0.5 if forget $(-\sin x)$

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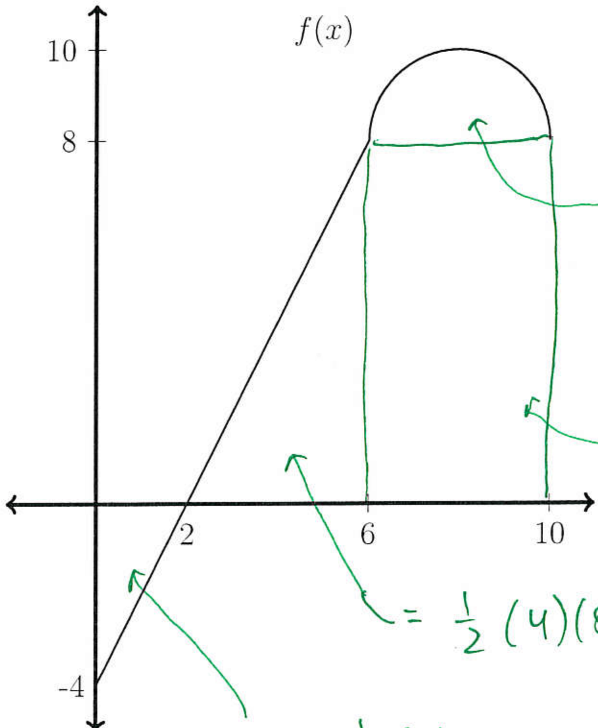
7. (5 points) Use geometry to calculate $\int_0^{10} f(x)\,dx$, where $f(x)$ is the function with the graph given below:



$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi$

$= 4 \cdot 8 = 32$

$= \frac{1}{2}(4)(8) = 16$

Area $= \frac{1}{2}(4)(2)$ (Negative!)

$= 4$

Area $= 32 + 16 + 2\pi - 4$

$= \boxed{44 + 2\pi}$

8. (5 points) Suppose you had to calculate

$$\int \frac{3x^3 \sec^{2}(x^3)}{5+\tan(x^3)}\,dx$$

by hand. What approach (integration by parts, u-substitution, etc) would you take to solving it? For this problem, I do NOT want you to get an answer. I want you to

(a) Explain (in words) *how* you would go about solving it

(b) Show the first few steps of your method (with calculation)

(c) State whether you think your approach will work or not, and why.

(a) Many answers My instinct is to do $u=(5+\tan(x^3))$ and try u-subs b/c have the $\sec^2(x^3)$ term

(b) $$\int \frac{\cancel{3}x^{\cancel{3}\,1}\,\cancel{\sec^2(x^3)}}{u}\cdot\frac{du}{\cancel{3x^2}\,\cancel{\sec^2(x^3)}}$$

$u=5+\tan(x^3)$

$du=\sec^2(x^3)\cdot 3x^2\,dx$

$\frac{du}{3x^2\sec^2(x^3)}=dx$

$$=\int \frac{x}{u}\,du$$

(c) Doesn't seem to work b/c one x did not cancel. Maybe try a int. by parts and split the x and $3x^2$ into two pieces?

9. (5 points) (a) Explain, in your own words, what the Fundamental Theorem of Calculus (FTC) says.

(b) Explain why the FTC is so important for calculating the area under a curve. In particular, you should mention *other* ways to calculate/approximate the area under a curve, and what about the FTC is so much easier than other methods.

(a) The FTC says that integration and differentiation are opposite operation, so they cancel each other. Thus you can calculate an integral by doing an anti derivative, and if you do $\frac{d}{dx}\int f(x)$, you just get back $f(x)$ as your result.

(b) The other methods are to do a Riemann sum (which is just approximation, not exact), or do the limit as $n \to \infty$, which is super complicated and long, or use geometry, which won't always work.

Comparatively, an antiderivative is pretty easy and calculates very fast.

Extra Credit(1 point) If $\int_1^4 f(x)\,dx = 3$, $\int_1^{13} g(x)\,dx = 6$, and $\int_4^{13} g(x)\,dx = 10$, calculate the value of the expression below (or explain why it cannot be done):

$$\int_4^1 (5f(x) + 2g(x))\,dx$$

$\Downarrow$

$\int_1^4 g(x) = 6 - 10 = -$

$= -\int_1^4 5f(x) - \int_1^4 2g(x)$

$= -5(3) - 2(-4)$

$= \boxed{-7}$