Quiz 4, Calculus 2 – Regular

Dr. Adam Graham-Squire, Spring 2020

Name: Key

1. (3 points) Find the interval and radius of convergence of $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{\sqrt[3]{n}\,5^n}$

Ratio Test: $\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}} \; x^{n+1}}{\sqrt[3]{n+1} \; \cancel{5^{n}}\,5} \cdot \frac{\sqrt[3]{n} \cdot \cancel{5^n}}{\cancel{(-1)^n} \; \cancel{x^n}} \right|$

$= \lim_{n\to\infty} \left| \frac{x}{5} \cdot \sqrt[3]{\frac{n}{n+1}} \right|$ goes to 1

$= \left| \frac{x}{5} \right| < 1$

$= |x| < 5 \Rightarrow$ $\boxed{\text{radius of conv} = 5}$

$\Rightarrow -5 < x < 5$ check

alternating!

$x = 5 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}} \cdot \frac{\cancel{5^n}}{\cancel{5^n}} = \sum \frac{(-1)^n}{\sqrt[3]{n}}$ converges by A.S.T.

$x = -5 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}} \cdot \frac{(-5)^n}{(5^n)}$

$= \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{\sqrt[3]{n}} \cdot \frac{5^n}{5^n} = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

diverges! p-series w/ $p = \frac{1}{3} < 1$.

$\boxed{\text{interval} = (-5, 5]}$

2. (3 points) Find the Taylor polynomial, up to the fourth term, for $f(x) = \sqrt{x}$ centered at $a = 1$ (in other words, find $T_3(x)$). Note: the general form for a Taylor series is $\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$.

NO need to multiply out $\Rightarrow$

$$T_3(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{6}(x-1)^3$$

$$= 1 + \frac{1}{2}(x-1) - \frac{\left(\frac{1}{4}\right)}{2}(x-1)^2 + \frac{\left(\frac{3}{8}\right)}{6}(x-1)^3$$

$$\boxed{= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{3}{48}(x-1)^3}$$

$f(x) = x^{1/2}$ $\qquad f(1) = 1$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\qquad f'(1) = \frac{1}{2}$

$f''(x) = \frac{-1}{4}x^{-3/2}$ $\qquad f''(1) = \frac{-1}{4}$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\qquad f'''(1) = \frac{3}{8}$

3. (4 points) (a) Find a power series representation for $\int \sin(x^2)\,dx$. (Hint: use the Maclaurin series for $\sin(x)$).

(b) Briefly explain what is the benefit of using a power series to calculate the integral of $\sin(x^2)$.

(a) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ $\quad (x^2)^{2n+1} = x^{4n+2}$

$\sin(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$

$\Rightarrow \int \sin(x^2)\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\,dx$

$$\boxed{= \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)!\,(4n+3)} + C}$$

(b) You can integrate $\sin(x^2)$ by hand b/c need to do substitution $x^2 = u$, but there is no x out in front so it doesn't work.