Quiz 3, Calculus 2 – Regular

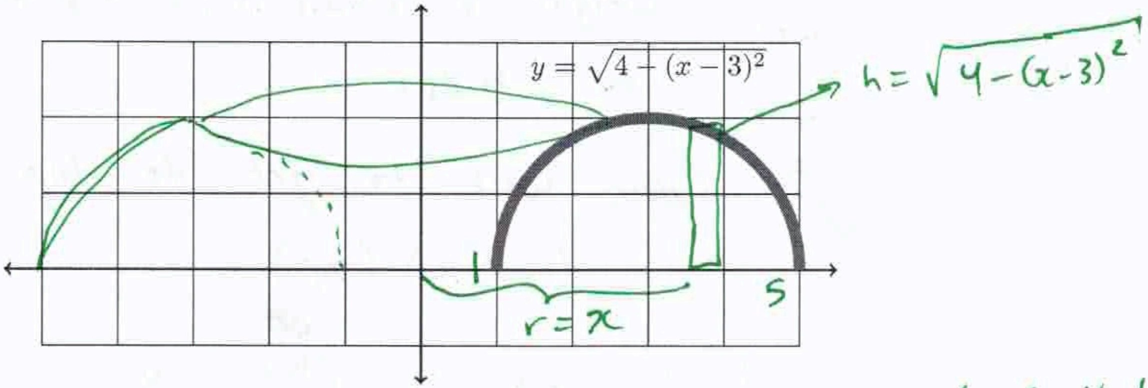
Dr. Adam Graham-Squire, Spring 2020

Name: Key

1. (3 points) Consider solid created by taking the area between the curve $y = \sqrt{4-(x-3)^2}$ and the x-axis, and rotating about the y-axis (see graph below). Answer the following questions about this volume of revolution problem:

   (a) Explain why it is better to use cylindrical shells to calculate the volume than the washer/disc method.

   (b) Set up, but do NOT integrate, an integral to calculate the volume described above (using shells).



(a) Two reasons: ① To do a disc, you would do it horizontally which is one part of the curve to another. That won't work.

✓

② The "thickness" for a washer is a Δy, so you would need to rewrite the expression ~~for~~ [illegible] as $x = f(y)$, which won't work well b/c of the $\sqrt{\ }$.

↓

give 0.5x for explanatio that says nothing, bu tries

(b) $2\pi r h = 2\pi(x)(\sqrt{4-(x-3)^2})$

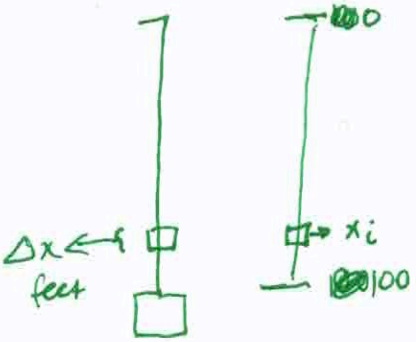
$$\Rightarrow \boxed{\int_1^5 2\pi x\sqrt{4-(x-3)^2}\,dx}$$

✓✓

2. (3 points) A 2000 pound elevator is suspended by a 100 foot rope that weighs 5 pounds per foot. Set up, but do not integrate, an integral to represent the amount of work necessary to lift the elevator 30 feet up.

· force to lift elevator is

2000 lbs × (30 feet)   0.5



0.5

· force for chunk of rope at height x_i with thickness Δx

force = weight of chunk is (Δx feet)×(5 lbs/ft)

w = 5Δx

0.5

work to get to top is

$(5\Delta x)(x_i)$

⇒ Total work is $\int_0^{100} 5x\,dx$ ✓

0.5

for only 30 feet is $\int_{70}^{100} 5x\,dx + 2000\times 30$ ⇒ b/c it is like lifting the x between 70 and 100

or $\int_{70}^{100} (5x + 2000)\,dx$

weight of rope is 5(100-x)

distance = Δx

$\int_0^{100} 5(100-x)\,\Delta x \Rightarrow \boxed{\int_0^{30} 5(100-x)\,dx}$ +2000

3. (4 points) Determine if the series converges or not. If the series converges, calculate the sum (if possible).

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-8)^n}{5(6^{n+1})}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{2}{n+2}\right)$

$$r \to = \frac{-4}{3}$$

$$(a) = \sum_{n=0}^{\infty} \frac{(-8)^n}{5 \cdot 6 \cdot (6^n)} = \sum_{n=0}^{\infty} \frac{1}{30} \cdot \left(\frac{-8}{6}\right)^n$$

is geometric with $r = \frac{-4}{3}$, $\left|\frac{-4}{3}\right| = \frac{4}{3} > 1$

so it <u>diverges</u>.

(can also use test for divergence.)

$$(b) \quad \overbrace{\overset{n=1}{\left(\frac{2}{1} - \cancel{\frac{2}{3}}\right)} + \overset{n=2}{\left(\frac{2}{2} - \cancel{\frac{2}{4}}\right)} + \overset{n=3}{\left(\cancel{\frac{2}{3}} - \cancel{\frac{2}{5}}\right)} + \left(\cancel{\frac{2}{4}} - \frac{2}{6}\right) + \left(\cancel{\frac{2}{5}} - \right.}^{S_5}$$

$$\Rightarrow S_n = \frac{2}{1} + \frac{2}{2} - \frac{2}{n+1} - \frac{2}{n+2}$$

$$\text{Sum} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(2 + 1 - \overset{\nearrow 0}{\frac{2}{n+1}} - \overset{\nearrow 0}{\frac{2}{n+2}}\right)$$

$$= \boxed{3} \qquad \boxed{\text{converges to } 3}$$