Quiz 2, Calculus 2

Dr. Adam Graham-Squire, Spring 2020

Name: Key

1. (~~6~~ 7 points) For each of the integrals below, give a brief explanation of how you would approach it and why, given the different methods of integration that we now know (substitution, integration by parts, trigonometric substitution, partial fractions, improper integrals, etc). Some integrals may have multiple answers, and some may not be integrable with any of the methods we have learned (and thus you would need to use Maple to get an approximate integral)–if so, you should say that. Note: you do NOT have to integrate the functions! It may help to do a couple of the first steps, though, to see if your method works.

2

1.5 for one but not both

(a) $\displaystyle\int_0^2 \frac{x-2}{x^2-1}\,dx$ → factor to $(x-1)(x+1)$ ⇒ discontinuity at $x=1$

⇒ $\boxed{\text{improper integral}}$, need to split up as

$$\lim_{n\to 1^-}\int_0^n \frac{x-2}{x^2-1}\,dx + \lim_{n\to 1^+}\int_n^2 \frac{x-2}{x^2-1}\,dx$$

Also, since $\frac{d}{dx}(x^2-1) = 2x$ is not a multiple of $x-2$, Need to use $\boxed{\text{partial fractions}}$ to break up the integrand

1.

(b) $\displaystyle\int_0^{0.1} \frac{x}{\sqrt{1-4x^2}}\,dx$

The $1-4x^2$ makes it seem like you could do $\boxed{\text{trig substitution}}$, but the easiest option is to do $\boxed{u\text{-substitution}}$ with $u = 1-4x^2$

$\Rightarrow du = -8x$

to get rid of x in numerator

✓

(c) $\displaystyle\int_0^2 \frac{1}{\sqrt{x^2+9}}\,dx$

This is a clear trig substitution problem with

$x = 3\tan\theta$

b/c of the $\sqrt{x^2+9}$ on the bottom

(d) $\int_0^2 e^{(x^3)}\,dx$ This is not integrable by hand b/c if you do $u = x^3$, you get an x^2 term popping out. Need to use Maple.

(e) $\int_0^2 \cos^5 x\,dx$ can do trig. subs here. Let $u = \sin x$
$du = \cos x\,dx$

$\int_0^2 (\cos x)(\cos^2 x)(\cos^2 x)$

$\int_0^2 \cos x\,(1-\sin^2 x)(1-\sin^2 x)$ etc.

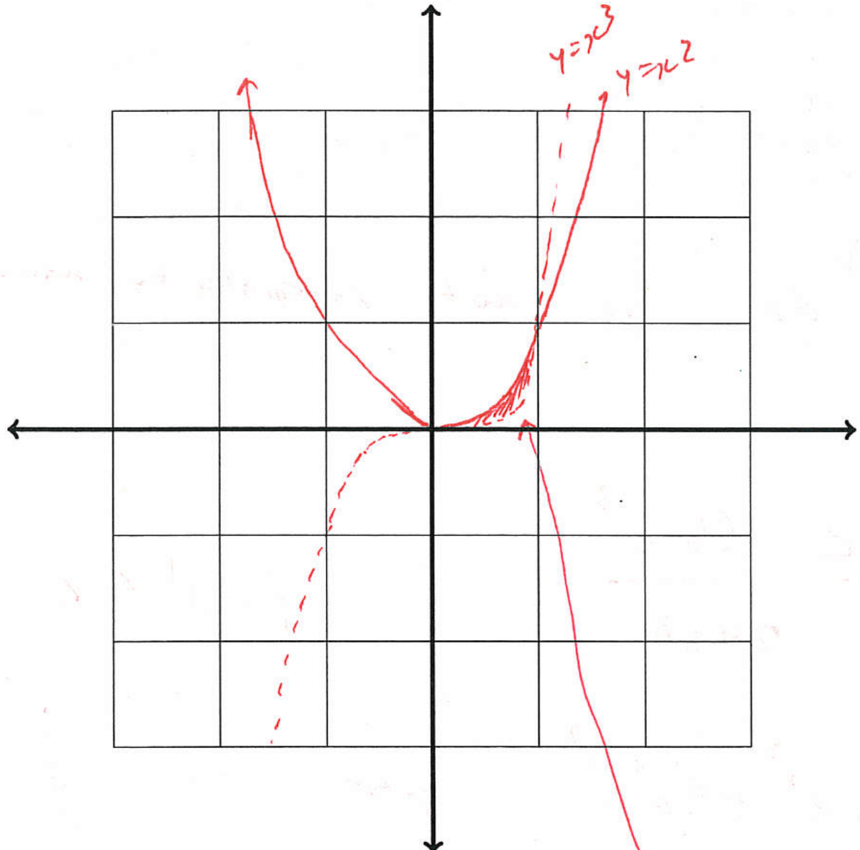
(f) $\int_1^2 x \ln(\sqrt{x})\,dx$ This will be integration by parts

because you can let ~~$u = x$~~
$u = \ln(\sqrt{x})$ and $dv = x\,dx$

$\downarrow$ when do du, the ln goes away

$\Downarrow$ gives you a polynomial, $\left(\frac{x^2}{2}\right)$ which is not so bad.

2. (2 points) Set up (but do not integrate) an integral to calculate the area completely enclosed by the functions $y = x^2$ and $y = x^3$. Sketch a graph of the area to help explain your integral:



$x^2 = x^3$

$0 = x^3 - x^2$

$0 = x^2(x - 1)$

$0 = x^2 (x - 1)$

$\Rightarrow 0 = x, \quad 1 = x$

That is the area, x^2 on top, x^3 on bottom, from 0 to 1

$$\Rightarrow \int_0^1 (x^2 - x^3)\, dx$$

9:30