Quiz 1, Calculus 2

Dr. Adam Graham-Squire, Spring 2020

7:45 $\Rightarrow$ 30 min

Name: Key

1. (3 points) (a) Calculate the Riemann sum to approximate the area under the curve for the function $f(x) = x^2$, from $x = 0$ to $x = 3$, using 5 subintervals and evaluating at *right* endpoints (that is, calculate R_5). Show/explain your work.

   (b) Is your answer from (a) an *over*estimate or an *under*estimate of the actual area under the curve? Use a quick sketch of the graph to explain your reasoning.

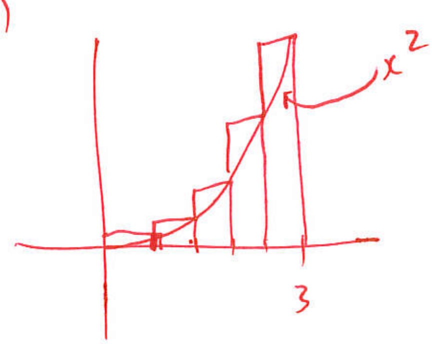
$\Delta x = \frac{3}{5}$

(a) $\frac{3}{5}\left(f(\frac{3}{5}) + f(\frac{6}{5}) + f(\frac{9}{5}) + f(\frac{12}{5}) + f(3)\right)$

$= \frac{3}{5}\left(\frac{9}{25} + \frac{36}{25} + \frac{81}{25} + \frac{144}{25} + 9\right)$

$= \frac{3 \cdot 270}{5 \cdot 25} + \frac{27}{5} = \cancel{8.28}\ \boxed{11.88}$

(b)



Over estimate b/c the rectangles are above the curve.

2. (2 points) Use the Fundamental Theorem of Calculus to calculate

$$\frac{d}{dx}\int_4^{\ln(x)} \sin(t^3)\,dt$$

$$= \boxed{\sin\left((\ln(x))^3\right) \cdot \frac{1}{x}}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

3. (5 points) Below, the first few steps are done for using the limit definition to calculate a definite integral, evaluating at the *right* endpoint. Answer the questions below. You can use the back side of this sheet if you need more space.

$$\begin{aligned}
\int_0^3 x^2\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x & (1)\\
&= \lim_{n\to\infty} \frac{3}{n}\left[f\left(\frac{3}{n}\right)+f\left(\frac{6}{n}\right)+f\left(\frac{9}{n}\right)+\cdots+f\left(\frac{3n}{n}\right)\right] & (2)\\
&= \lim_{n\to\infty} \frac{3}{n}\left[\left(\frac{3}{n}\right)^2+\left(\frac{6}{n}\right)^2+\left(\frac{9}{n}\right)^2+\cdots+\left(\frac{3n}{n}\right)^2\right] & (3)\\
&= \lim_{n\to\infty} \frac{3}{n}\left(\frac{3}{n}\right)^2\left[1^2+2^2+3^2+\cdots+n^2\right] & (4)\\
&= \lim_{n\to\infty} \left(\frac{3}{n}\right)^3\left[\sum_{i=1}^{n} i^2\right] & (5)
\end{aligned}$$

✓ (a) Explain where the $\frac{3}{n}$ comes from (between lines 1 and 2).

✓ (b) Explain what is happening between lines 3 and 4. Be clear as to why you get that result.

✓✓ (c) Use the magic formula $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to finish calculating the limit.

✓ (d) Use antiderivatives and the Fundamental Theorem of Calculus (the evaluation theorem) to calculate $\int_0^3 x^2\,dx$ to double-check your answer from (3c).

(a) The $\frac{3}{n}$ is the width of the rectangles, b/c the whole interval is 3 units long, divided by n rectangles.

(b) A $\left(\frac{3}{n}\right)^2$ is being factored out from each term, so you are left with some integer squared. For example

$$\left(\frac{6}{n}\right)^2 = \left(\frac{2\cdot 3}{n}\right)^2 = 2^2\left(\frac{3}{n}\right)^2$$

so when you factor out $\left(\frac{3}{n}\right)^2$, you are left with

2^2.

(c) $\lim_{n\to\infty} \left(\frac{3}{n}\right)^3 \left(\frac{n(n+1)(2n+1)}{6} \right)$

$= \lim_{n\to\infty} \frac{\cancel{27}\,9}{n^3} \left(\frac{(n^2+n)(2n+1)}{\cancel{6}\,2} \right)$

$= \lim_{n\to\infty} \frac{9\,(2n^3+3n^2+n)}{2n^3}$

$= \lim_{n\to\infty} \frac{18n^3}{2n^3} + \frac{27n^2}{2n^3} + \frac{9n}{2n^3}$

$= \lim_{n\to\infty} 9 + \underbrace{\frac{27}{2n}}_{\to 0} + \underbrace{\frac{9}{2n^2}}_{\to 0}$

$= \boxed{9}$

(d) $\int_0^3 x^2\,dx = \frac{1}{3}x^3 \Big|_0^3 = \frac{1}{3}(3^3) = \boxed{9}$