Quiz 1, Calculus 2

Dr. Adam Graham-Squire, Spring 2020

Name: ______________________________

1. (3 points) (a) Calculate the Riemann sum to approximate the area under the curve for the function $f(x) = x^2$, from $x = 0$ to $x = 3$, using 5 subintervals and evaluating at *right* endpoints (that is, calculate R_5). Show/explain your work.

   (b) Is your answer from (a) an *over*estimate or an *under*estimate of the actual area under the curve? Use a quick sketch of the graph to explain your reasoning.

2. (2 points) Use the Fundamental Theorem of Calculus to calculate

$$\frac{d}{dx}\int_{4}^{\ln(x)} \sin(t^3)\,dt$$

3. (5 points) Below, the first few steps are done for using the limit definition to calculate a definite integral, evaluating at the *right* endpoint. Answer the questions below. You can use the back side of this sheet if you need more space.

$$\int_0^3 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \tag{1}$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[f\left(\frac{3}{n}\right) + f\left(\frac{6}{n}\right) + f\left(\frac{9}{n}\right) + \cdots + f\left(\frac{3n}{n}\right)\right] \tag{2}$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\left(\frac{3}{n}\right)^2 + \left(\frac{6}{n}\right)^2 + \left(\frac{9}{n}\right)^2 + \cdots + \left(\frac{3n}{n}\right)^2\right] \tag{3}$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{3}{n}\right)^2 \left[1^2 + 2^2 + 3^2 + \cdots + n^2\right] \tag{4}$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)^3 \left[\sum_{i=1}^{n} i^2\right] \tag{5}$$

(a) Explain where the $\frac{3}{n}$ comes from (between lines 1 and 2).

(b) Explain what is happening between lines 3 and 4. Be clear as to why you get that result.

(c) Use the magic formula $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to finish calculating the limit.

(d) Use antiderivatives and the Fundamental Theorem of Calculus (the evaluation theorem) to calculate $\int_0^3 x^2\,dx$ to double-check your answer from (3c).