Test 3A - MTH 1420

Dr. Graham-Squire, Spring 2013

Name: Key

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I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Read the questions carefully, and make sure you answer all parts.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones are not allowed on this test. Calculators are allowed on the first 3 questions, however you should still show all of your work to receive full credit. If you are asked to integrate something, I expect you to integrate it by hand unless otherwise specified. Calculators are not allowed on the last 5 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 8. Total Points = 75.

Corrections on #2

Calculators are okay

1. (10 points) Do the following sequences converge or diverge? If it converges, find the limit. Make sure to show your work and justify your answer.

   (a) $\left\{1+\dfrac{(-1)^n}{n}\right\}$

$$\lim_{n\to\infty} 1 + \frac{(-1)^n}{n} = 1 + 0 = \boxed{1}$$

   (b) $\left\{\dfrac{n^2}{n+3}-\dfrac{n^2}{n+4}\right\}$

$$\lim_{n\to\infty} \frac{n^2}{n+3} - \frac{n^2}{n+4} = \lim_{n\to\infty} \frac{n^2(n+4)-n^2(n+3)}{(n+3)(n+4)}$$

$$= \lim_{n\to\infty} \frac{n^3+4n^2-n^3-3n^2}{(n+3)(n+4)}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+7n+12} = \boxed{1}$$

2. (10 points) Do the following series converge or diverge? Show your work and state which test(s) you use to justify your answer.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi}{n^{(1/e)}}$

$\frac{1}{e} \approx 0.4 < 1$

change to $n=1$

So $\pi \sum \frac{1}{n^{(1/e)}}$ is a p-series with $p<1$

$\Rightarrow$ $\boxed{\text{diverges by p-series}}$

change to $n=1$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{5^{2n-1}(n^2)}{(2n-1)!}$

Ratio test: $\lim_{n\to\infty} \left| \frac{5^{2n+1}(n+1)^2}{(2n+1)!} \cdot \frac{(2n-1)!}{n^2 \cdot 5^{2n-1}} \right|$

$= \lim_{n\to\infty} \left| \frac{5^2}{(2n)(2n+1)} \cdot \left(\frac{n+1}{n}\right)^2 \right|$

goes to 1

$= 0$

$\Rightarrow$ $\boxed{\text{converges}}$ absolutely

3. (10 points) Do the following series converge or diverge? If the series converges, find the exact value of the sum. If the series converges and you <u>cannot</u> find the exact value of the sum, use an approximation method to approximate the exact value of the sum to 0.01. Show your work and state which test(s) you use to justify your answer.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n n}{3^n}$

Ratio test: $\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(n+1)}{3^{n+1}} \cdot \frac{3^n}{(-1)^n n} \right|$

$= \lim_{n\to\infty} \left| \frac{1}{3} \cdot \frac{n+1}{n} \right| = \frac{1}{3}$

converges. Can't find exact sum b/c not geometric or telescoping. Use Alt. Series approx.:

$0 - \frac{1}{3} + \frac{2}{9} - \frac{3}{27} + \frac{4}{81} - \frac{5}{243} + \frac{6}{721}$

less than 0.01

$\Rightarrow \boxed{\text{sum} \approx -0.19}$

(b) $\sum_{n=4}^{\infty} \frac{2n-3}{n^2-3n}$

like $\sum \frac{1}{n}$

do LCT: $\lim_{n\to\infty} \left| \frac{\frac{2n-3}{n^2-3n}}{\frac{1}{n}} \right| = \lim_{n\to\infty} \frac{2n^2-3n}{n^2-3n} = 2$

$\Rightarrow$ $\boxed{\text{diverges}}$ b/c $\sum \frac{1}{n}$ diverges (harmonic)

(can also do Integral Test)

4. (10 points) Do the following series converge or diverge? If the series converges, find the exact value of the sum. If the series converges and you cannot find the exact value of the sum, use an approximation method to approximate the exact value of the sum to 0.01. Show your work and state which test(s) you use to justify your answer.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{5^{n+2}}{2^{3n+1}}$ $= \frac{25}{2} + \frac{125}{16} + \frac{625}{128} + \cdots$ ✓✓✓

geometric with $a = \frac{25}{2}$, $r = \frac{5}{8} < 1$

$\Rightarrow$ converges to $\dfrac{\frac{25}{2}}{1-\frac{5}{8}} = \frac{25}{2} \cdot \frac{8}{3} = \boxed{\frac{100}{3}}$

✓✓,

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \ln(n)$

$\lim_{n\to\infty} \ln(n) = \infty$

$\Rightarrow \lim_{n\to\infty} (-1)^n \ln(n)$ dne, $\neq 0$

$\Rightarrow$ $\boxed{\text{diverges by test for divergence}}$

5. (10 points) Do the following series converge or diverge? If the series converges, state if it converges absolutely or conditionally.

(a) $\sum_{n=0}^{\infty} \frac{\sin^3 n}{n!}$

$$|\sin^3 n| \leq 1$$

$$\Rightarrow \sum \left|\frac{\sin^3 n}{n!}\right| \leq \sum \frac{1}{n!}$$

Ratio test on:

$$\lim_{n\to\infty} \left|\frac{1}{(n+1)!} \cdot \frac{n!}{1}\right| = \lim_{n\to\infty} \frac{1}{n+1} = 0$$

$\Rightarrow$ Converges

$\Rightarrow \sum \frac{\sin^3 n}{n!}$ converges absolutely

(b) $\sum_{n=0}^{\infty} \cos(n) - \cos(n+1)$

$$= (\cos(0) - \cancel{\cos(1)}) + (\cancel{\cos(1)} - \cancel{\cos(2)}) + (\cancel{\cos(2)} - \cos(3)) + \cdots$$

<u>Telescoping</u>

$$\Rightarrow S_n = \cos(0) - \cos(n+1)$$

$$\lim_{n\to\infty} S_n = 1 - \lim_{n\to\infty} (n+1) \rightarrow \text{does not exist}$$

$\Rightarrow$ $\boxed{\text{series diverges}}$

6. (10 points) For the following power series, find the radius and interval of convergence.

(a) $\sum_{n=0}^{\infty} \frac{(x+3)^n}{\sqrt{2n+1}}$

Ratio test:

$$\lim_{n\to\infty} \left| \frac{(x+3)^{n+1}}{(x+3)^n} \cdot \frac{\sqrt{2n+3}}{\sqrt{2n+1}} \right|$$

goes to 1

$$= |x+3| < 1$$

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$\boxed{\Rightarrow \text{Radius of convergence} = 1}$

$-1 < x+3 < 1$

$-3 \quad -3 \quad -3$

$-4 < x < -2$

✓

Check: -4

get $\sum \frac{(-1)^n}{\sqrt{2n+1}}$ converges

by A.S.T. b/c $\lim_{n\to\infty} \frac{1}{\sqrt{2n+1}} \to 0$ decreasing.

✓

at -2 get $\sum \frac{1}{\sqrt{2n+1}}$

diverges b/c it is like* $\sum \frac{1}{\sqrt{n}}$, p-series w/ $p = 1/2 < 1$

$\boxed{\text{Interval} = [-4, 2)}$

✓

*LCT: $\lim_{n\to\infty} \frac{\frac{1}{\sqrt{2n+1}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{\frac{n}{2n+1}} = \sqrt{\frac{1}{2}}$

(b) $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2 (n!)}$

R.T. $\lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{(n+1)^2 (n+1)!} \cdot \frac{n! \, n^2}{3^n x^n} \right|$

$$= \lim_{n\to\infty} \left| \frac{3x}{n+1} \cdot \left(\frac{n}{n+1}\right)^2 \right|$$

goes to 0 goes to 1

✓✓

$$= 0$$

$\Rightarrow$ converges for all x

✓ Radius $= \infty$

✓ Interval $= (-\infty, \infty)$.

7. (10 points) For the following functions, find

(i) a power series representation for the function

(ii) the radius of convergence for the power series.

(a) $f(x) = \dfrac{x^2}{7 - x^2} = \dfrac{x^2}{7}\left(\dfrac{1}{1-\frac{x^2}{7}}\right)$

$= \dfrac{x^2}{7} \sum_{n=0}^{\infty} \left(\dfrac{x^2}{7}\right)^n$

(i) $= \boxed{\sum_{n=0}^{\infty} \dfrac{x^{2n+2}}{7^{n+1}}}$

✓✓✓✓

if $\left|\dfrac{x^2}{7}\right| < 1$ ✓

$|x|^2 < 7$

(ii) $\boxed{|x| < \sqrt{7}}$

(b) $f(x) = \ln(1 - 2x) = \int \left(\dfrac{d}{dx}\ln(1-2x)\right) dx$

$= \int \dfrac{1}{1-2x}(-2)\, dx$

$= -2 \int \sum_{n=0}^{\infty} (2x)^n \, dx$

$= -2 \int \sum_{n=0}^{\infty} 2^n x^n \, dx$

$= -2 \sum_{n=0}^{\infty} 2^n \dfrac{x^{n+1}}{n+1} + C$

$= \boxed{\sum_{n=0}^{\infty} -(2^{n+1}) \dfrac{x^{n+1}}{n+1}} + C$

✓✓✓✓

if $|2x| < 1$

$\boxed{|x| < \frac{1}{2}}$ ✓

$C = 0$ b/c $\ln(1 - 2(0)) = 0$ and $\sum -\dfrac{(2^{n+1})\, 0^{n+1}}{n+1} = 0$ ✓

8. (5 points) Does the series converge or diverge? Show your work and state which test(s) you use to justify your answer.

$$\sum_{n=0}^{\infty} \frac{1}{2^n + \sin n} < \sum_{n=0}^{\infty} \frac{1}{2^n - 1}$$

b/c $-1 \le \sin n \le 1$

L.C.T. with $\frac{1}{2^n}$

$$\lim_{n\to\infty} \frac{\frac{1}{2^n-1}}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{2^n}{2^n-1} = 1$$

and $\sum \frac{1}{2^n}$ is geom. with $r = \frac{1}{2} < 1$

$\Rightarrow$ $\boxed{\sum \frac{1}{2^n + \sin n} \text{ converges by comparison}}$

Extra Credit(up to 3 points) Write either a 1 or a 3 into the space below to request how many points you want for extra credit. If you put a 1 you are guaranteed 1 point. If you put a 3 and less than half the class also puts a 3, then you get 3 points. If more than half the class puts a 3, you get zero.

3	1
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