Minitest 4A - MTH 1420

Dr. Graham-Squire, Spring 2013

12:37
12:52
15

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Read the questions carefully, and make sure you answer all parts.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 3 questions, however you should still show all of your work to receive full credit. If you are asked to integrate something, I expect you to integrate it by hand unless otherwise specified. Calculators are not allowed on the last 5 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 5. Total Points = 40.

Calculators are okay

1. (8 points) Find the Taylor series for $f(x) = \ln(2+x)$ centered at $a = -1$. Simplify your expression to simplest terms. Hint: Find the first 5 or 6 derivatives and then look for a pattern to represent $f^{(n)}(a)$.

$f'(x) = \frac{1}{2+x} = (2+x)^{-1}$

$f''(x) = -(2+x)^{-2}$

$f'''(x) = -(-2)(2+x)^{-3}$

$f^{(4)}(x) = 2(-3)(2+x)^{-4}$

$f^{(5)}(x) = -6(-4)(2+x)^{-5}$

$\Rightarrow f(-1) = 0$

$f'(-1) = 1$

$f''(-1) = -1 = -1!$

$f'''(-1) = 2 = 2!$

$f^{(4)}(-1) = -6 = -3!$

$f^{(5)}(-1) = 24 = 4! \Rightarrow f^{(n)}(-1) = (-1)^{n+1}(n-1)!$

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$a = 1$

$f^{(n)}(a) = (-1)^{n+1}(n-1)!$

for $n \geq 1$

$$\Rightarrow T(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n-1)!}{n!}(x+1)^n$$

$$\boxed{\ln(2+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x+1)^n}$$

2. (8 points) (a) Find the Maclaurin series representation for $\cos(x^2)$.

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos(x^2) = \sum_{n=1}^{\infty} (-1)^n \frac{(x^2)^{2n}}{(2n)!}$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$$

(b) How many terms of the series you found in (a) are needed to get an error of less than 0.0001 on the interval [-0.7,0.7]? Justify your reasoning.

Max will occur at endpoints. By the Alt. Series Estimation theorem, need to find when $\frac{(0.7)^{4n}}{(2n)!} < 0.0001$

$n=0 \Rightarrow 1$

$n=1 \Rightarrow \frac{0.7^4}{2} = 0.12005$

$n=2 \Rightarrow \frac{0.7^8}{4!} = 0.0024$

$n=3 \Rightarrow \frac{0.7^{12}}{6!} = 0.000019$ is less than 0.0001

$\Rightarrow$ Need to add up the first <u>3 terms</u>.

NO CALCULATORS

Name: Key

Test A

3. (8 points) Match the equation with the graph.

(a) $9x^2 - 6y^2 + 9z^2 = 36$ (iii)

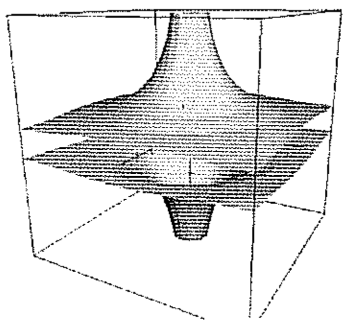
(b) $x^2 - 25y^2 = 9z^2 + 36$ (iv)

(c) $y^2 + z^2 = 25$ (ii)

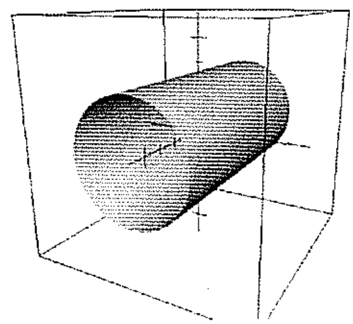
(d) $10y = 10z^2 - x^2$ (v)

(e) $x^2 + y^2 = \frac{1}{z^2}$ (i)

(e)



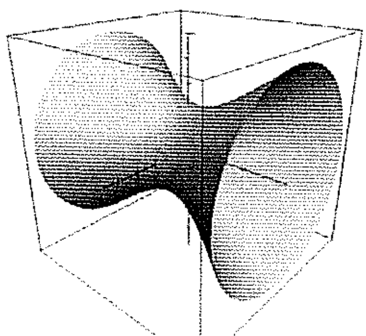
(i)



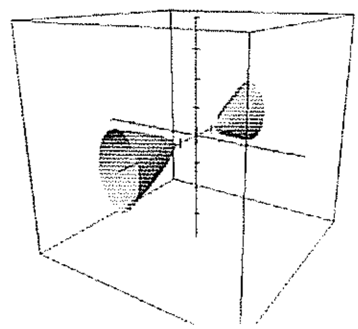
(c)

(ii)

(a)



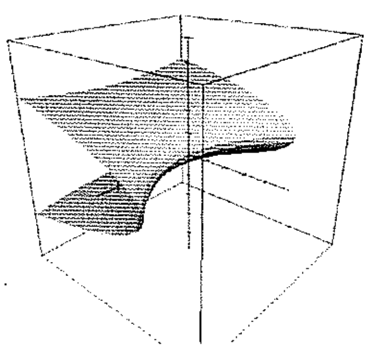
(iii)



(b)

(iv)

(d)



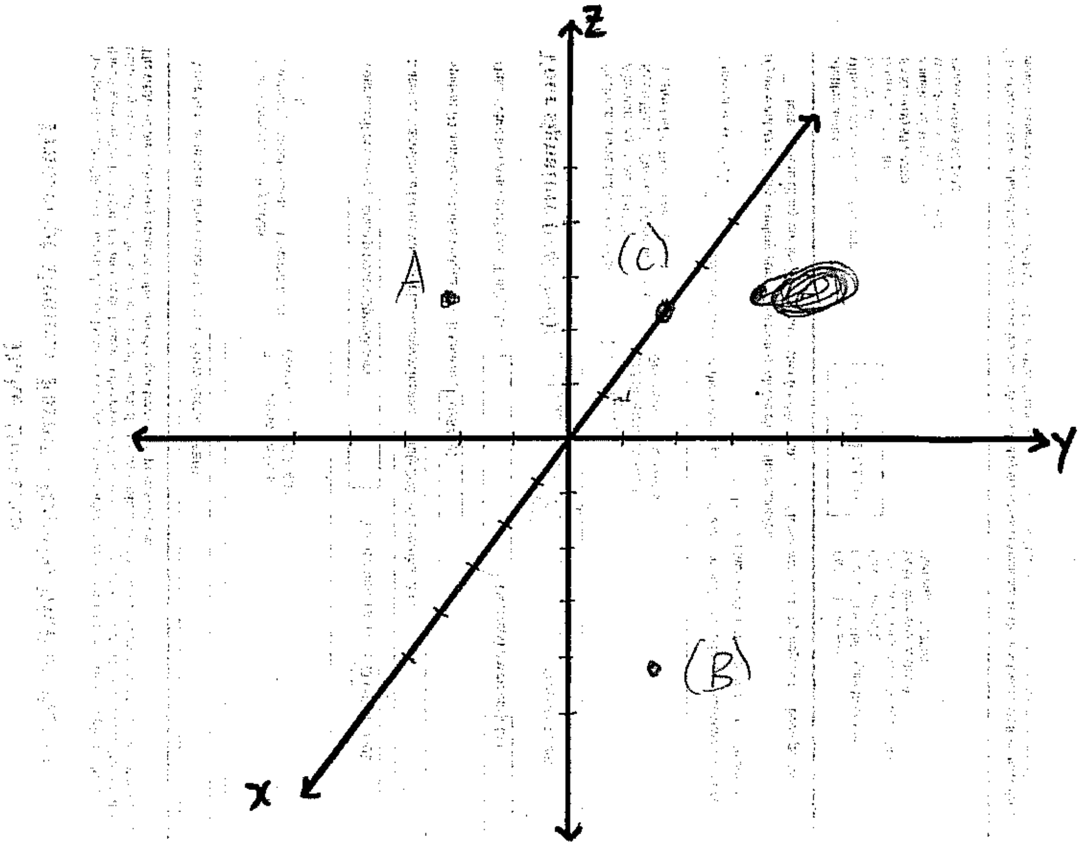
(v)

5. (8 points) Plot the following points on the given set of 3-D axes:

   (A) Rectangular coordinates: (-1,-3, 2)

   (B) Cylindrical coordinates: $(4, \frac{\pi}{4}, -2)$.

   (C) Spherical coordinates: $(3, \pi, \frac{\pi}{2})$



Extra Credit(1 point) Calculate the first ~~three~~ 3 terms of the Maclaurin series for $(e^x)(\frac{1}{1-x})$.

$$\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}+\cdots\right)\left(1+x+x^2+x^3+\cdots\right)$$

$$= 1+(x+x)+\left(x^2+x^2+\frac{x^2}{2}\right)+\cdots$$

$$= 1+2x+\frac{5}{2}x^2+\cdots$$

36

4. (8 points) (a) Convert the equation $x^2 + y^2 + z^2 = \cancel{16}$ 36 from rectangular to spherical coordinates. Explain why the equation in spherical coordinates makes sense.

$\rho^2 = 36 \Rightarrow \boxed{\rho = 6}$

Makes sense b/c $\rho = 6$ is radius of sphere, and θ and ϕ can be anything.

$x^2+y^2+z^2 = 6^2$ is sphere of radius 6.

(b) Convert the point (-2$\sqrt{3}$, -2, 5) from rectangular to cylindrical coordinates.

$r^2 = x^2+y^2 \Rightarrow r^2 = (-2\sqrt{3})^2 + (-2)^2 \Rightarrow r^2 = 16 \Rightarrow r = \pm 4$

$\tan\theta = \frac{y}{x} = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \theta$

$\Rightarrow \theta = 30° = \frac{\pi}{6}$

$(-2\sqrt{3}, -2)$ is in Quadrant 3 $\Rightarrow r = 4,\ \theta = \frac{7\pi}{6}$

or

$r = -4,\ \theta = \frac{\pi}{6}$

$$\boxed{\left(4, \frac{7\pi}{6}, 5\right)}$$

Minitest 4B - MTH 1420

Dr. Graham-Squire, Spring 2013

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DIRECTIONS

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2. Read the questions carefully, and make sure you answer all parts.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the last 2 questions, however you should still show all of your work to receive full credit. If you are asked to integrate something, I expect you to integrate it by hand unless otherwise specified. Calculators are not allowed on the first 3 questions, and once you turn in the non-calculator portion you cannot go back to it.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 5. Total Points = 40.

1. (8 points) Match the equation with the graph.

   (a) $x^2 - 25y^2 = 9z^2 + 36$ (iv)

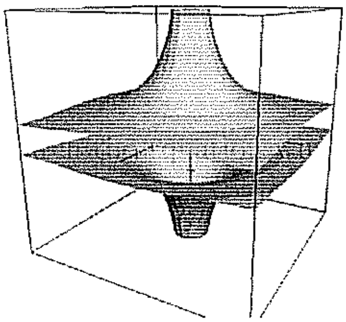
   (b) $x^2 + y^2 = \dfrac{1}{z^2}$ (i)

   (c) $y^2 + z^2 = 25$ (ii)

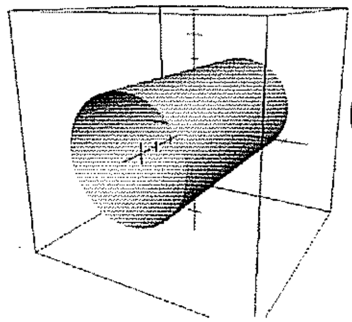
   (d) $10y = 10z^2 - x^2$ (v)

   (e) $9x^2 - 6y^2 + 9z^2 = 36$ (iii)

(b)



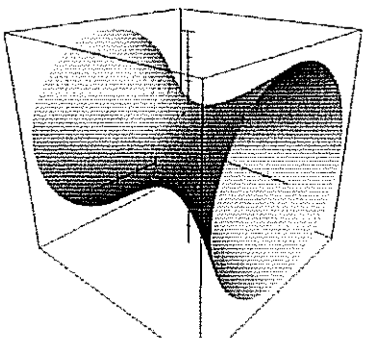
(i)



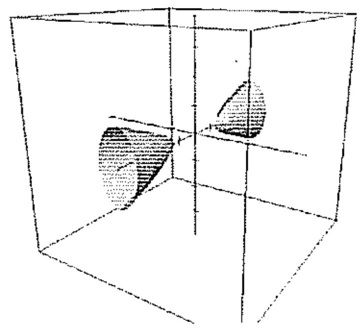
(c)

(ii)

(e)



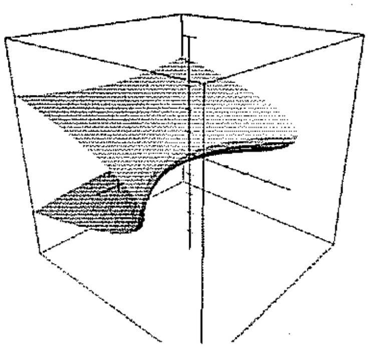
(iii)



(a)

(iv)

(d)



(v)

−1 per wrong answer.

2. (8 points) (a) Convert the point (-2, -2$\sqrt{3}$, 5) from rectangular to cylindrical coordinates.

$\cancel{\rho^2 = (-2)^2 + (-2\sqrt{3})^2 + 5^2 \Rightarrow \rho^2 = 4 + 12 + 25 = 41 \quad \rho = \sqrt{41}}$

$r^2 = (-2)^2 + (-2\sqrt{3})^2 \Rightarrow r^2 = 16 \Rightarrow r = \pm 4$

$\tan\theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3} \Rightarrow \theta = \arctan(\sqrt{3}) = \frac{\pi}{3}$ ← reference angle

Since $(-2, -2\sqrt{3})$ is in Quad III, correct answer is

$\boxed{(4, \frac{4\pi}{3}, 5)}$ or $(-4, \frac{\pi}{3}, 5)$

4

(b) Convert the equation $x^2 + y^2 + z^2 = 9$ from rectangular to spherical coordinates. Explain why the equation in spherical coordinates makes sense.

$\rho^2 = x^2 + y^2 + z^2 \Rightarrow \rho^2 = 9 \Rightarrow \boxed{\rho = 3}$ is the equation

4

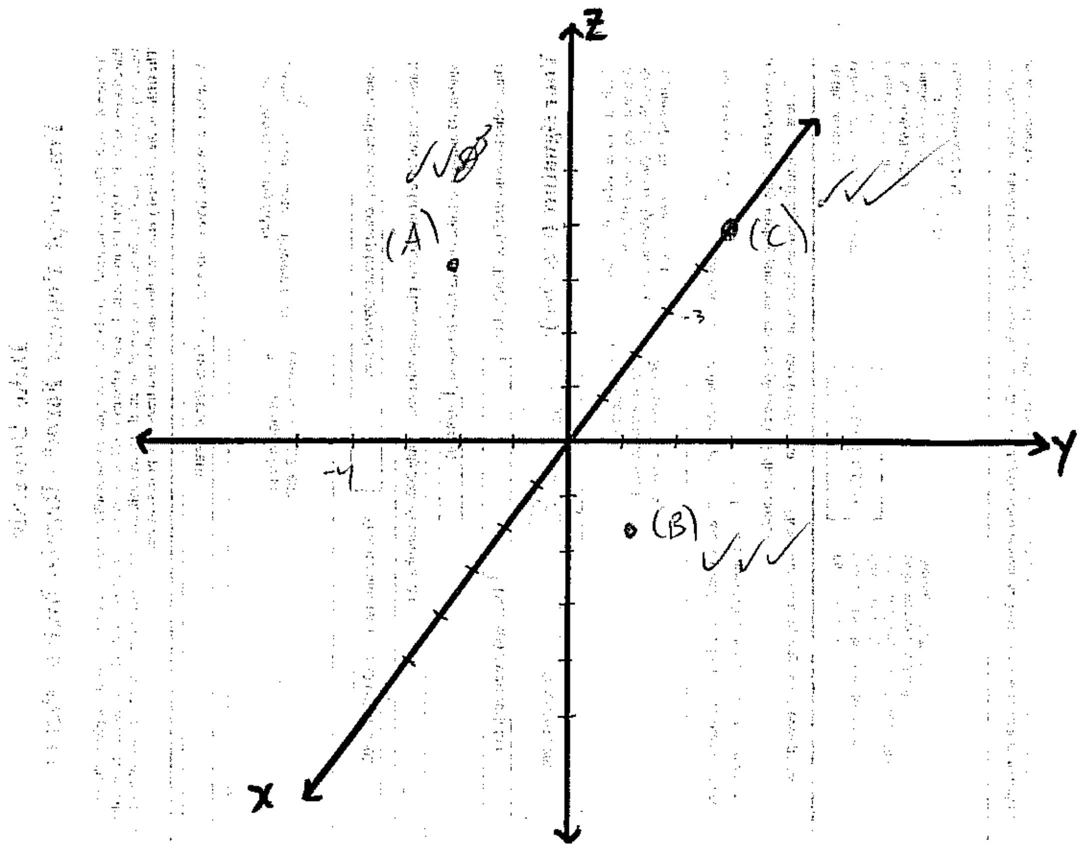
Makes sense b/c $x^2 + y^2 + z^2 = 9$ is a sphere with radius 3, and ρ corresponds to distance from the origin (i.e. radius). Since there ~~are~~ is no θ or ϕ in the equation, they can be anything which is what gives you all points on the sphere.

3. (8 points) Plot the following points on the given set of 3-D axes:

(A) Rectangular coordinates: (-3, -4, 1)

(B) Cylindrical coordinates: $(3, \frac{\pi}{4}, 1)$.

(C) Spherical coordinates: $(5, \pi, \frac{\pi}{2})$



Calculators are okay

Name: Key

Test B

4. (8 points) (a) Find the Maclaurin series representation for $\cos(x^2)$.

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n}}{(2n)!} = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}}$$

$r = \infty$

(b) How many terms of the series you found in (a) are needed to get an error of less than 0.0001 on the interval [-0.7,0.7]? Justify your reasoning.

Need to know when $\frac{x^{4n}}{(2n)!} \le 0.0001$ for $x = 0.7$

$n=0$ get $\frac{0.7^0}{0!} = 1$

$n=1$ get $\frac{0.7^4}{2!} = 0.12$

$n=2$ get $\frac{0.7^8}{4!} = \cancel{0.0024}\ 0.024$

only need to add up first 3 terms

$n=3$ get $\frac{0.7^{12}}{6!} = \cancel{0.0023}\ 0.000019 < 0.0001$

By A.S.T. estimation test

~~$n=4$ get $\frac{0.7^{16}}{8!} = 0.0004$~~

5. (8 points) Find the Taylor series for $f(x) = \ln(2+x)$ centered at $a = -1$. Simplify your expression to simplest terms. Hint: Find the first 5 or 6 derivatives and then look for a pattern to represent $f^{(n)}(a)$.

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^n \quad \checkmark$$

$$\left.\begin{aligned}
f(x) &= \ln(2+x) \\
f'(x) &= \frac{1}{2+x} = (2+x)^{-1} \\
f''(x) &= -(2+x)^{-2} \\
f'''(x) &= 2(2+x)^{-3} \\
f^{(4)}(x) &= -6(2+x)^{-4}
\end{aligned}\right\} @3$$

$$f^{(5)}(x) = 24(2+x)^{-5}$$

$$\Rightarrow f^{(n)}(x) = (n-1)!\,(2+x)^{-n}\,(-1)^{n+1} \quad \checkmark$$

$$\Rightarrow f^{(n)}(-1) = (n-1)!\,(1)\,(-1)^{n+1} = (-1)^{n+1}(n-1)! \quad \checkmark$$

$$\Rightarrow \ln(2+x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(n-1)!}{n!}(x+1)^n \quad \checkmark$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}(x+1)^n} \quad \checkmark$$

Extra Credit(1 point) Calculate the first three terms of the Maclaurin series for $(e^x)(\frac{1}{1-x})$.

$$\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}+\cdots\right)\left(1+x+x^2+x^3+\cdots\right)$$

$$= 1+(x+x)+\left(x^2+x^2+\frac{x^2}{2}\right)+\cdots$$

$$= \boxed{1+2x+2.5x^2}+\cdots$$