Mini-Test 1 Review Key

Dr. Graham-Squire, Spring 2013

1. Suppose a particle on a line has velocity $v(t) = t(2-t)$, for t-values between 0 and 4.
   (a) Explain in words the difference between the *displacement* of the particle and the *total distance traveled* from $t = 0$ to $t = 4$.

   Ans: The displacement is the difference in location from where the particle started to where it ended, you find it by calculating $\int_0^4 2(2-t)dt$. The total distance traveled is when you calculate both the forward and backward distances as positive. You find this by calculating $|\int_0^2 2(2-t)dt| + |\int_2^4 2(2-t)dt|$

   (b) Now calculate each of them using integrals.

   Ans: The displacement is equal to -16/3, the total distance is equal to 8.

2. (a) Using midpoints as your evaluation points, find an approximation for $\int_3^5 (2x-7)dx$ using 4 rectangles.

   Ans: 2

   (b) Use the Fundamental Theorem of Calculus to evaluate $\int_3^5 (2x-7)dx$. How close is your answer to 2(a)? Explain.

   Ans: 2. This is identical to what we got in part (a) because the approximation using the midpoint gives you the exact answer, since the over and under approximation cancel each other out.

3. Evaluate the indefinite integral $\int [x(x+2)^2 + \sec^2 x]dx$.

   Ans: $\frac{1}{4}x^4 + \frac{2}{3}x^3 + 2x^2 + \tan x + C$. You do not need to use substitution or integration by parts for this one, just multiply out the integrand.

4. Find $h'(x)$ if $h(x) = \int_0^{e^{2x}} \sin^3 t\, dt$.

   Ans: $2e^{2x}\sin^3(e^{2x})$. Note that you need to show your work and explain how you use the fundamental theorem of calculus to come up with the answer (just getting the answer is not enough).

5. Evaluate the integrals.

(a) $\displaystyle\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Ans: 4. Use substitution with $u = \sqrt{x}$.

(b) $\displaystyle\int x^2\sqrt[3]{1-x}\,dx$

Ans: $-\frac{3}{4}(1-x)^{4/3} + \frac{6}{7}(1-x)^{7/3} - \frac{3}{10}(1-x)^{10/3} + C$. Use substitution with $u = 1-x$, you will need to rewrite to substitute in $x = 1-u$ also. OR, use integration by parts- you will have to do it a couple of times to get to the right answer. If you do integration by parts, you should get an answer that looks similar to mine, but is not exactly the same. If you simplify and multiply things out, they will come out to be equal but you should not worry about doing that.

(c) $\displaystyle\int x\ln x\,dx$

Ans: $\frac{x^2\ln x}{2} - \frac{x^2}{4} + C$. Use integration by parts.