Quiz 5B, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

1. (4 points) Find the radius and interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{x^n}{(-4)^n\sqrt{n+1}}$$

Ratio test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(-4)^{n+1}(\sqrt{n+2})} \cdot \frac{(-4)^n\sqrt{n+1}}{x^n} \right|$

$= \lim_{n\to\infty} \frac{|x|}{4} \cdot \sqrt{\frac{n+1}{n+2}}$

$= \frac{|x|}{4}$

Need $\frac{|x|}{4} < 1$

$\Rightarrow |x| < 4 \Rightarrow \boxed{R=4}$

$-4 < x < 4$

Check endpoints:

at $x=4$ get $\sum \frac{(-1)^n}{\sqrt{n+1}}$ converges by A.S.T. b/c $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$

at $x=-4$ get $\sum \frac{1}{\sqrt{n+1}}$ diverges by comparison with $\sum \frac{1}{n^{1/2}}$, p-series with $p<1$

$\Rightarrow$ interval is $-4 < x \le 4$, $\boxed{(-4,4]}$

2. (2 points) Does the series $\sum_{n=1}^{\infty} \frac{n^2}{e^{(2/n)}}$ converge or diverge? Justify your reasoning and state which test you use.

As $n\to\infty$, $n^2 \to \infty$ and $e^{2/n} \to e^{2/\infty} = e^0 = 1$

$\Rightarrow \lim_{n\to\infty} \frac{n^2}{e^{(2/n)}} = \infty$ So series diverges by test for divergence.

3. (2 points) Is the series absolutely convergent, conditionally convergent, or divergent? State what convergence or divergence test(s) you use, and show your work.

$$\sum_{n=5}^{\infty}(-1)^n \frac{\sqrt[3]{n}}{n-4}$$

by A.S.T. $\frac{\sqrt[3]{n}}{n-4} \to 0$ as $n \to \infty$, and is decreasing.

So series converges.

$\left|(-1)^n \frac{\sqrt[3]{n}}{n-4}\right| = \frac{\sqrt[3]{n}}{n-4}$, $\sum_{n=5}^{\infty} \frac{\sqrt[3]{n}}{n-4}$ diverges. compare to $\sum \frac{1}{n^{2/3}}$ by

Limit Comparison: $\lim_{n\to\infty} \frac{\sqrt[3]{n}}{n-4} \cdot \frac{n^{2/3}}{1} = \lim_{n\to\infty} \frac{n}{n-4} = 1$, so they

do the same thing, and $\sum \frac{1}{n^{2/3}}$ is a p-series with $p<1$.

$\Rightarrow$ $\boxed{\text{conditionally convergent}}$

4. (2 points) Is the series absolutely convergent, conditionally convergent, or divergent? State what convergence or divergence test(s) you use, and show your work.

$$\sum_{n=1}^{\infty} \frac{(-15)^n}{n!}$$

Ratio test: $\lim_{n\to\infty} \left| \frac{(-15)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-15)^n} \right|$

$= \lim_{n\to\infty} \frac{15}{n+1} = 0 < 1$

$\boxed{\text{converges absolutely by Ratio test}}$

Quiz 5A, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

3:23
3:31
8 ⇒ give 20/25 minutes.

1. (2 points) Does the series $\sum_{n=1}^{\infty} \frac{n^3}{e^{(3/n)}}$ converge or diverge? Justify your reasoning and state which test you use.

$$\lim_{n\to\infty} \frac{n^3}{e^{(3/n)}} = \frac{\infty^3}{e^0} = \frac{\infty}{1} = \infty \neq 0$$

✓✓ diverges by Test for divergence.

2. (2 points) Is the series absolutely convergent, conditionally convergent, or divergent? State what convergence or divergence test(s) you use, and show your work.

$$\sum_{n=8}^{\infty} (-1)^n \frac{\sqrt[3]{n}}{n-7}$$

A.S.T.

Consider $\left\{\frac{\sqrt[3]{n}}{n-7}\right\}$. ✓✓ Since $n^{1/3}$ has lower power than $n-7$, sequence is decreasing, also $\lim_{n\to\infty} \frac{\sqrt[3]{n}}{n-7} = 0$ b/c power on top is lower.

$\Rightarrow$ converges by A.S.T.

X However $\sum_{n=8}^{\infty} \frac{\sqrt[3]{n}}{n-7} > \sum_{n=8}^{\infty} \frac{n^{1/3}}{n} = \sum_{n=8}^{\infty} \frac{1}{n^{2/3}}$ diverges b/c it is a p-series with $p = 2/3 < 1$

$\Rightarrow$ not abs. converges

$\Rightarrow$ ✓✓ Converges conditionally

3. (2 points) Is the series absolutely convergent, conditionally convergent, or divergent? State what convergence or divergence test(s) you use, and show your work.

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$$

Ratio test: $\left|\frac{(-10)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-10)^n}\right|$

$= \left|\frac{-10}{n+1}\right|$

$\lim_{n\to\infty} \left|\frac{-10}{n+1}\right| = 0 \Rightarrow$ $\boxed{\text{converges absolutely by Ratio Test}}$

4. (4 points) Find the radius and interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{x^n}{(-3)^n\sqrt{n+1}}$$

R.T.: $\left|\frac{x^{n+1}}{(-3)^{n+1}\sqrt{n+2}} \cdot \frac{(-3)^n\sqrt{n+1}}{x^n}\right|$

$= \left|\frac{x}{-3} \cdot \sqrt{\frac{n+1}{n+2}}\right|$

$\lim_{n\to\infty} \left|\frac{x}{-3} \cdot \sqrt{\frac{n+1}{n+2}}\right| = \frac{|x|}{3}$

$\Rightarrow$ converges when

$\frac{|x|}{3} < 1 \Rightarrow |x| < 3$

Radius of conv = 3

$|x| < 3 \Rightarrow -3 < x < 3$

Check $x = -3$, get $\sum \frac{1}{\sqrt{n+1}}$ is p-series with $p = 1/2$, diverges.

Check $x = 3$, get $\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$ converges by A.S.T. b/c $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} \to 0$ and is decreasing.

$\Rightarrow$ $\boxed{\text{Interval is } -3 < x \leq 3}$