Quiz 4A, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

1. (2 points) Does the sequence $\left\{\frac{\sin n}{n^{1/2}}\right\}$ converge or diverge? Justify your reasoning.

$$-1 \le \sin n \le 1 \Rightarrow \frac{-1}{n^{1/2}} \le \frac{\sin n}{n^{1/2}} \le \frac{1}{n^{1/2}}$$

$$\Rightarrow \lim_{n\to\infty} \frac{-1}{n^{1/2}} \le \lim_{n\to\infty} \frac{\sin n}{n^{1/2}} \le \lim_{n\to\infty} \frac{1}{n^{1/2}}$$

$$0 \le \text{"} \le 0$$

$\Rightarrow \frac{\sin n}{n^{1/2}}$ converges to 0 by Squeeze Theorem.

2. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work. If the series converges, find the sum.

$$\sum_{n=1}^{\infty} n^{-1/4} = \sum \frac{1}{n^{1/4}}$$

p-series with $p = \frac{1}{4} < 1$

$\Rightarrow$ diverges.

3. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work.

$$\sum_{n=1}^{\infty} \frac{4n}{(2n^2+1)^3}$$

$u = 2x^2 + 1$

$du = 4x\, dx$

Integral test $\int_1^{\infty} \frac{4x}{(2x^2+1)^3}\, dx$

$= \lim_{b\to\infty} \int_3^b \frac{1}{u^3}\, du$

$= \lim_{b\to\infty} \left(\frac{u^{-2}}{-2} \Big|_3^b \right) = \lim_{b\to\infty} \frac{b^{-2}}{-2} - \frac{3^{-2}}{-2} = 0 + \frac{1}{18}$ converges by integral test.

4. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work. If the series converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{5^n}{4^{2n}} = \sum_{n=1}^{\infty} \frac{5^n}{16^n}$$

geometric with $a = \frac{5}{16}$ $r = \frac{5}{16}$

Converges to $\dfrac{\frac{5}{16}}{1-\frac{5}{16}} = \frac{5}{16} \cdot \frac{16}{11} = \boxed{\frac{5}{11}}$

5. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work.

$$\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)$$

$\lim_{n\to\infty} \left(1 - \frac{3}{n}\right) = 1 - 0 = 1 \neq 0$

So diverges by test for divergence.

Quiz 4B, Calculus 2

Dr. Graham-Squire, Spring 2013

2:10
2:15
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5

Name: Key

1. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work.

$$\sum_{n=1}^{\infty}\left(2-\frac{1}{n^2}\right)$$

$a_n = 2 - \frac{1}{n^2}$

$\lim_{n\to\infty} \left(2-\frac{1}{n^2}\right) = 2 - 0 = 2 \neq 0$

$\Rightarrow$ $\boxed{\text{diverges by Test for divergence}}$

2. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work. If the series converges, find the sum.

$$\sum_{n=1}^{\infty}\frac{3^n}{2^{3n}} = \sum_{n=1}^{\infty} \frac{3^n}{(2^3)^n} = \sum_{n=1}^{\infty} \left(\frac{3}{8}\right)^n$$

$\Rightarrow$ geometric. $a = \frac{3}{8}$

$r = \frac{3}{8} < 1$

$\Rightarrow$ converges to $\dfrac{\frac{3}{8}}{1-\frac{3}{8}} = \dfrac{\frac{3}{8}}{\frac{5}{8}} = \boxed{\frac{3}{5}}$

3. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work. If the series converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{4n^3}{n^4+7}$$

$u = x^4+7$

$du = 4x^3$

x	u
1	8
∞	∞

Integral test: $\int_1^\infty \frac{4x^3}{x^4+7}\,dx$

$= \int_8^\infty \frac{1}{u}\,du$

$= \lim_{b\to\infty} \ln|u|\Big|_8^b$

$= \lim_{b\to\infty} \ln|b| - \ln 8 = \infty \Rightarrow \boxed{\text{diverges}}$

can also do limit comparison w/ $\frac{1}{n}$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\left(\frac{4n^3}{n^4+7}\right)}$ $\lim_{n\to\infty} \frac{n^4+7}{4n^4} = 4$

$\Rightarrow$ diverges.

4. (2 points) Does the sequence $\left\{\frac{\cos n}{n^{1/5}}\right\}$ converge or diverge? Justify your reasoning.

$-1 \le \cos n \le 1 \Rightarrow \frac{-1}{n^{1/5}} \le \frac{\cos n}{n^{1/5}} \le \frac{1}{n^{1/5}}$

$\Rightarrow \lim_{n\to\infty} \frac{-1}{n^{1/5}} \le \lim_{n\to\infty} \frac{\cos(n)}{n^{1/5}} \le \frac{1}{n^{1/5}}$

$0 \le \lim_{n\to\infty} \frac{\cos n}{n^{1/5}} \le 0$

$\boxed{\text{converges to } 0}$ by squeeze thm!

5. (2 points) Does the series converge or diverge? State what convergence or divergence test you use, and show your work.

$$\sum_{n=1}^{\infty} n^{-7} = \sum_{n=1}^{\infty} \frac{1}{n^7}$$

p-series with $p = 7 > 1$

$\Rightarrow \boxed{\text{converges}}$