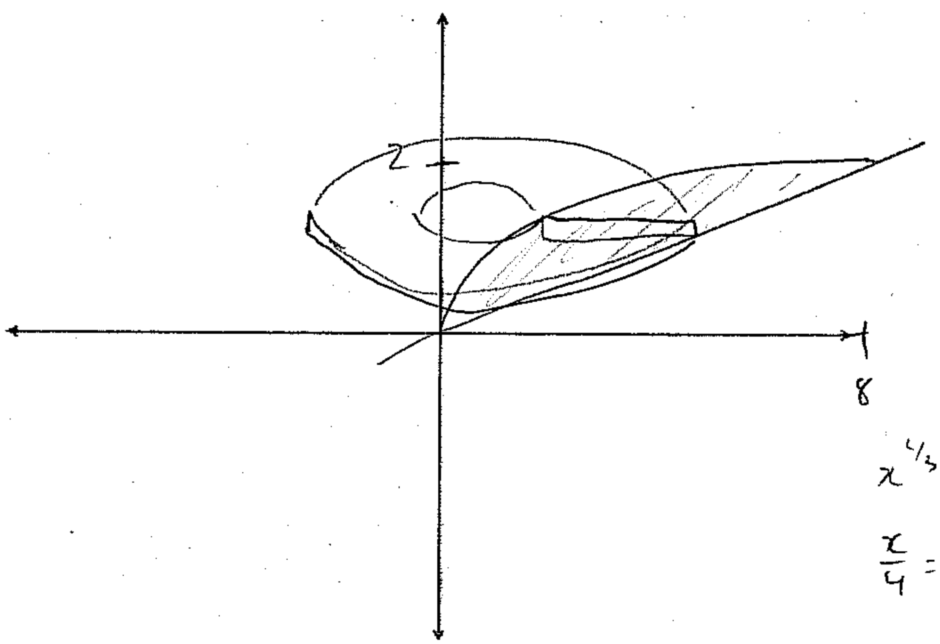
Quiz 3A, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

1. (5 points) Let A be the region in the first quadrant enclosed by the curves $x^{1/3} = y$ and $\frac{x}{4} = y$. Find the volume of the solid formed by rotating A about the y-axis. You should integrate it by hand, but you can use a calculator or Sage/Maple to check your work. Leave your answer in exact form, <u>not</u> a decimal approximation.



$x^{1/3} = \frac{x}{4}$

$x = \frac{x^3}{64}$

$x^3 - 64x = 0$

$x(x^2-64) = 0$

$x = 0,\ x = 8$

$x^{1/3} = y \Rightarrow y^3 = x$

$\frac{x}{4} = y \Rightarrow x = 4y$

$$\text{Volume} = \pi\int_0^2 \left((4y)^2 - (y^3)^2\right) dy$$

$$= \pi\int_0^2 (16y^2 - y^6)\, dy$$

$$= \pi\left(\frac{16}{3}y^3 - \frac{y^7}{7}\Big|_0^2\right)$$

$$= \pi\left(\frac{8\cdot 16}{3} - \frac{128}{7}\right) = \frac{\pi(7\cdot 8\cdot 16 - 128\cdot 3)}{21}$$

or

$$\int_0^8 2\pi x\left(x^{1/3} - \frac{x}{4}\right) dx$$

$$= 2\pi\int_0^8 \left(x^{4/3} - \frac{x^2}{4}\right) dx$$

$$= 2\pi\left(\frac{3}{7}x^{7/3} - \frac{x^3}{12}\Big|_0^8\right)$$

$$= 2\pi\left(\frac{3\cdot 128}{7} - \frac{64\cdot 8}{12}\right)$$

2. (5 points) (a) Set up an integral to calculate the arc length of the parametric curve $x = t^2$, $y = t^{5/2} + 4$, for $0 \leq t \leq 4$. Simplify the integrand, if possible.

(b) If you think you can integrate it by hand, explain how you would do it. If you think you cannot integrate it by hand, explain why.

(c) Use your calculator or Sage/Maple to evaluate the integral, **you do not need to integrate it completely by hand.** Round to the nearest 0.01.

$\frac{dx}{dt} = 2t$

$\frac{dy}{dt} = \frac{5}{2} t^{3/2}$

(a) $\int_0^4 \sqrt{4t^2 + \frac{25}{4} t^3}\, dt$

$= \int_0^4 t \sqrt{4 + \frac{25}{4} t}\, dt$

(b) Can integrate by hand. Either

(i) Let $u = 4 + \frac{25}{4} t$ and $\frac{u-4}{\left(\frac{25}{4}\right)} = t$ (ie. use substitution)

or (ii) integration by parts with $u = t$ $dv = \left(4 + \frac{25}{4} t\right)^{1/2}$

(c) On Sage, $\int_0^4 t \left(4 + \frac{25}{4} t\right)^{1/2} dt = 35.93$

Quiz 3B, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

8:24
8:33
9

1. (5 points) (a) Set up an integral to calculate the arc length of the parametric curve $x = t^2 - 7$, $y = t^{5/2}$, for $0 \le t \le 9$. Simplify ~~evaluate~~ the integrand, if possible.

   (b) If you think you can integrate it by hand, explain how you would do it. If you think you cannot integrate it by hand, explain why.

   (c) Use your calculator or Sage/Maple to evaluate the integral, **you do not need to integrate it completely by hand.** Round to nearest 0.01

(a) $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ $\qquad \frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = \frac{5}{2}t^{3/2}$

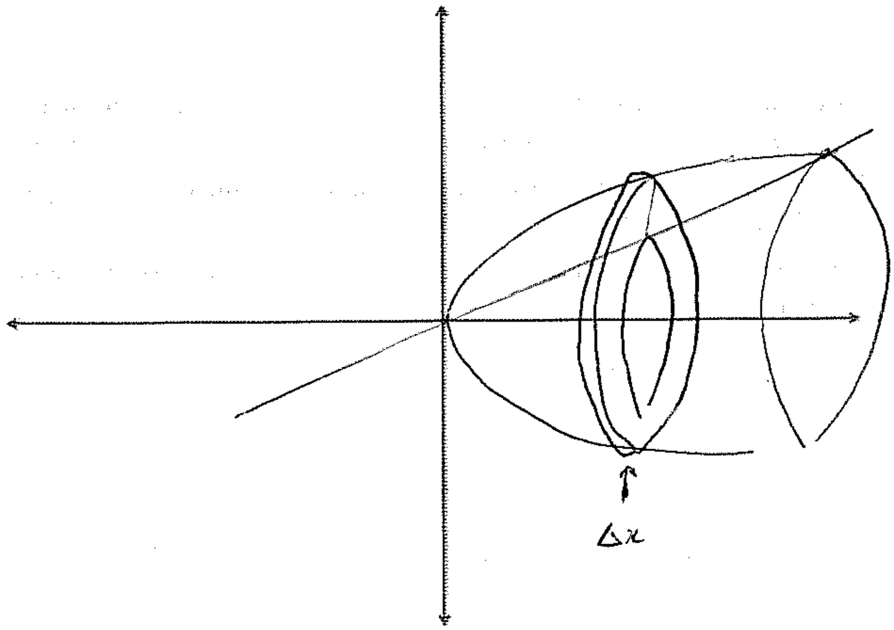
$= \int_0^9 \sqrt{(2t)^2 + \frac{25}{4}t^3}\, dt$

$= \int_0^9 \sqrt{t^2\left(4 + \frac{25}{4}t\right)}\, dt = \int_0^9 t\sqrt{4 + \frac{25}{4}t}$

(b) Can integrate by hand. Would have to do substitution with $u = 4 + \frac{25}{4}t$ or do integration by parts with $u = t$ and $v = \left(4 + \frac{25}{4}t\right)^{1/2}$

(c) 256.82

2. (5 points) Let A be the region in the first quadrant enclosed by the curves $x = y^2$ and $x = 3y$. Find the volume of the solid formed by rotating A about the x-axis. You should integrate it by hand, but you can use a calculator or Sage/Maple to check your work.



$\sqrt{x} = \frac{x}{3}$

$x = \frac{x^2}{9}$

$x^2 - 9x = 0$

$x(x-9) = 0$

outer radius $\Rightarrow$ $y = \sqrt{x}$

inner radius $\Rightarrow$ $y = \frac{x}{3}$

$$V \Rightarrow \pi \int_0^9 \left((\sqrt{x})^2 - \left(\frac{x}{3}\right)^2 \right) dx$$

$$= \pi \int_0^9 \left(x - \frac{x^2}{9} \right) dx$$

$$= \pi \left(\frac{x^2}{2} - \frac{x^3}{27} \Big|_0^9 \right)$$

$$= \pi \left(\frac{81}{2} - \frac{81 \cdot \cancel{9}}{\cancel{27}\,3} - 0 \right)$$

$$= \pi \left(\frac{81}{2} - \frac{54}{2} \right) = \boxed{\frac{27\pi}{2}}$$