# Quiz 2A, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

1. (6 points) Consider the definite integral

$$\int_0^2 \frac{x^3}{\sqrt{4-x^2}}\,dx.$$

   (a) Is the integral an improper integral or not? If it is, explain why and express it appropriately with a limit.

   (b) Use trigonometric substitution with $x = 2\sin\theta$ to simplify and evaluate the integral. If you can't do the trig substitution, you can solve it by another method for partial credit. Show your work!

(a) Yes! At $x=2$ there is a vertical asymptote b/c you are dividing by zero. Should be

$$\lim_{a\to 2^-}\int_0^a \frac{x^3}{\sqrt{4-x^2}}\,dx$$

(b)

$$\lim_{a\to 2^-}\int_0^{\sin^{-1}(\frac{a}{2})} \frac{(2\sin\theta)^3}{\sqrt{4-(2\sin\theta)^2}}\cdot 2\cos\theta\,d\theta$$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

| $x$ | $\theta$ |
|---|---|
| $0$ | $0$ |
| $a$ | $\sin^{-1}(\frac{a}{2})$ |

$$= \lim_{a\to 2^-}\int_0^{\sin^{-1}(\frac{a}{2})} \frac{8\sin^3\theta}{\sqrt{4(1-\sin^2\theta)}}\cdot 2\cos\theta\,d\theta$$

$$= \lim_{a\to 2^-}\int_0^{\sin^{-1}(\frac{a}{2})} \frac{8\sin^3\theta}{\cancel{2}\cancel{\cos\theta}}\cdot \cancel{2}\cancel{\cos\theta}\,d\theta$$

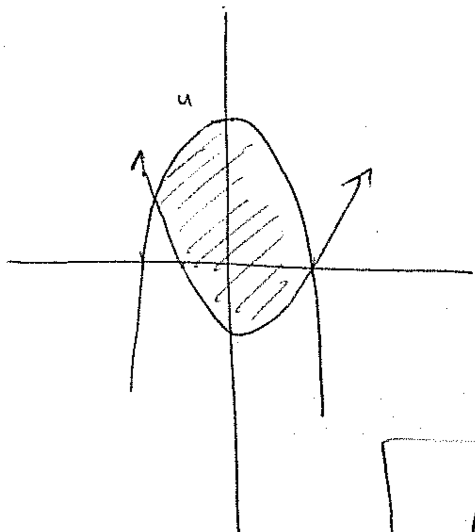
$$= 8\int_0^{\pi/2} \sin^3\theta\,d\theta$$

Now do $\sin^2\theta\cdot\sin\theta$, then write $(1-\cos^2\theta)$ for $\sin^2\theta$ and do u subst. with $u=\cos\theta$

can evaluate the limit.

2. (4 points) Set up but do not integrate a definite integral to find the area enclosed by the curves

$$f(x) = x^2 - x - 2 \qquad \text{and} \qquad g(x) = 4 - x^2.$$



$x^2 - x - 2 = 4 - x^2$ ← to find intersection points.

$2x^2 - x - 6 = 0$

$(2x+3)(x-2) = 0$

$x = \frac{-3}{2}$ or $x = 2$

$$\boxed{\int_{-3/2}^{2} \left[(4-x^2) - (x^2-x-2)\right] dx}$$

# Quiz 2B, Calculus 2

Dr. Graham-Squire, Spring 2013

8:46
8:54

**Name:** Key

1. (4 points) Set up but <u>do not integrate</u> a definite integral to find the area enclosed by the curves

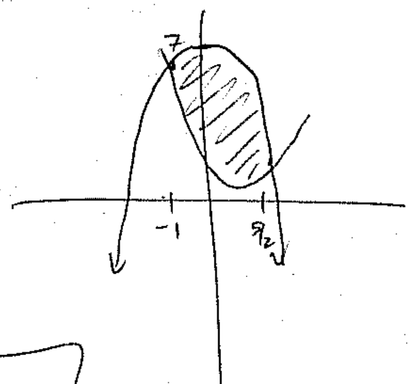
$$f(x) = x^2 - 3x + 2 \qquad \text{and} \qquad g(x) = 7 - x^2.$$

$x^2 - 3x + 2 = 7 - x^2$

$\Rightarrow 2x^2 - 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$

$\Rightarrow x = \frac{5}{2}, \; x = -1$




$$\boxed{\int_{-1}^{5/2} \left[ \left(7 - x^2\right) - \left(x^2 - 3x + 2\right) \right] dx}$$

2. (6 points) Consider the definite integral

$$\int_0^3 \frac{x^3}{\sqrt{9-x^2}}\,dx.$$

(a) Is the integral an improper integral or not? If it is, explain why and express it appropriately with a limit.

(b) Use trigonometric substitution with $x = 3\sin\theta$ to simplify and evaluate the integral. If you can't do the trig substitution, you can solve it by another method for partial credit. Show your work!

(a) Yes, it is improper because a $x=3$ there is a vertical asymptote. (Dividing by 0).

Need to do $\lim_{b\to 3}\int_0^b \frac{x^3}{\sqrt{9-x^2}}\,dx$

(b) $\lim_{b\to 3}\int_0^b \frac{x^3}{\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$\frac{b}{3} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{b}{3}\right)$

$\frac{0}{3} = \sin\theta$

$0 = \theta$

$$= \lim_{b\to 3}\int_0^{b/3} \frac{(3\sin\theta)^3}{\cancel{\sqrt{9-(3\sin\theta)^2}}}\cdot 3\cancel{\cos\theta}\, d\theta$$

$$= \lim_{b\to 3} 27\int_0^{b/3} \sin^3\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

| $\theta$ | $u$ |
|---|---|
| 1 | $\cos 1$ |
| 0 | 1 |

$$= \int_0^1 \sin\theta\,(1-\cos^2\theta)\,d\theta$$

$$= -\int_1^{\cos 1}(1-u^2)\,du$$

$$= -\left(u - \tfrac{1}{3}u^3\Big|_1^{\cos 1}\right) = \boxed{-\left(\cos 1 - \tfrac{1}{3}(\cos 1)^3 - \left(1-\tfrac{1}{3}\right)\right)}$$