Quiz 6A, Calculus 2

Dr. Graham-Squire, Spring 2013

1:32

1:35

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Name: Key

1. (3 points) (a) Use the Maclaurin series for e^x to express $x^2e^{(-x^3)}$ as a power series. Make sure to simplify your answer.

   (b) What will be the radius of convergence for the power series you found in part (a)?

(a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{(-x^3)} = \sum_{n=0}^{\infty} \frac{(-x^3)^n}{n!} \Rightarrow x^2 e^{(-x^3)}$

$$= x^2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{n!}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+2}}{n!}}$$

(b) $R = \infty$ (same as for e^x)

2. (3 points) Find the second Taylor polynomial ($T_2(x)$, which is the approximation up to the 3rd term) for the function $y = \ln x$ at $a = e$. Note that

$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{n!}(x-a)^i$$

$f(x) = \ln x$, $f(e) = 1$

$f'(x) = \frac{1}{x}$ $f'(e) = \frac{1}{e}$

$f''(x) = \frac{-1}{x^2}$ $f''(e) = \frac{-1}{e^2}$

$$\Rightarrow T_2(x) = \frac{1}{0!}(x-e)^0 + \frac{1}{e}\cdot\frac{(x-e)^1}{1!} + \left(\frac{-1}{e^2}\right)\frac{(x-e)^2}{2!}$$

$$\boxed{T_2(x) = 1 + \frac{x-e}{e} - \frac{(x-e)^2}{2e^2}}$$

3. (4 points) Match the equation to the graph:

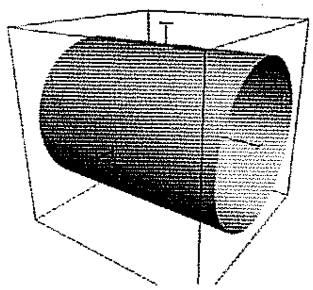
(a) $16 = x^2 + y^2$ (iv)

(b) $z = x^2$ (v)

(c) $z^2 + x^2 = 16$ (i)

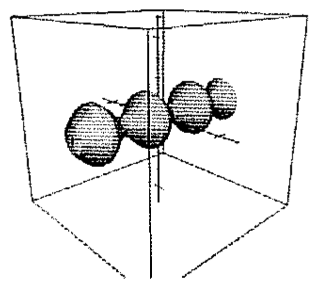
(d) $1 = -\frac{x^2}{25} - \frac{y^2}{16} + \frac{z^2}{9}$ (iii)

(e) $z^2 + y^2 = (2 + \sin(x))^2$ (ii)



(c)

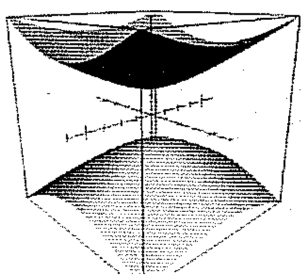
(i)



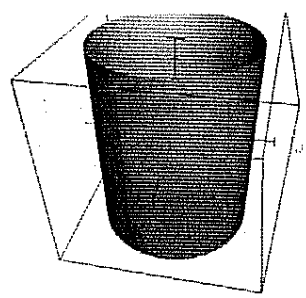
(e)

(ii)

(d)

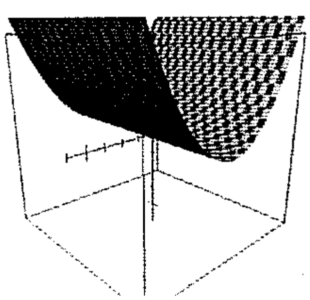


(iii)



(a)

(iv)



(b)

(v)

Quiz 6B, Calculus 2

Dr. Graham-Squire, Spring 2013

Name: Key

1. (3 points) Find the second Taylor polynomial ($T_2(x)$, which is the approximation up to the 3rd term) for the function $y = \ln x$ at $a = e$. Note that

$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{n!}(x-a)^i$$

$f(x) = \ln x$

$y' = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$$T_2(x) = \frac{f(e)}{0!}(x-e)^0 + \frac{f'(e)}{1!}(x-e) + \frac{f''(e)}{2!}(x-e)^2$$

$$= 1 + \frac{1}{e}(x-e) - \frac{1}{e^2} \cdot \frac{1}{2}(x-e)^2$$

$$= 1 + \frac{(x-e)}{e} - \frac{(x-e)^2}{2e^2}$$

2. (3 points) (a) Use the Maclaurin series for e^x to express $x^3 e^{(-x^2)}$ as a power series. Make sure to simplify your answer.

(b) What will be the radius of convergence for the power series you found in part (a)?

(a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{(-x^2)} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

$$x^3 e^{(-x^2)} = x^3 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!}}$$

(b) $|(-x^2)| < \infty$ (b/c e^x has radius $= \infty$)

$\Rightarrow \sqrt{x^2} < \sqrt{\infty}$

$x < \infty$

$\Rightarrow \boxed{\infty}$

3. (4 points) Match the equation to the graph:

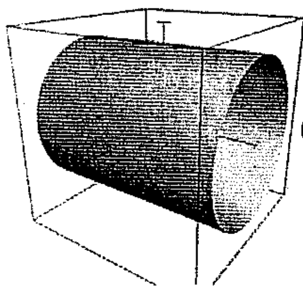
(a) $1 = -\frac{x^2}{25} - \frac{y^2}{16} + \frac{z^2}{9}$ (iii)

(b) $16 = x^2 + y^2$ (iv)

(c) $z = x^2$ (v)

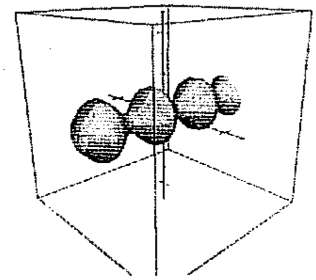
(d) $z^2 + y^2 = (2 + \sin(x))^2$ (ii)

(e) $z^2 + x^2 = 16$ (i)



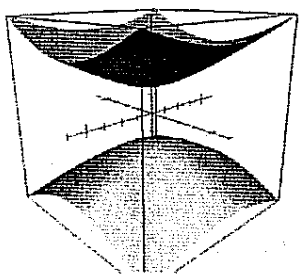
(e)

(i)



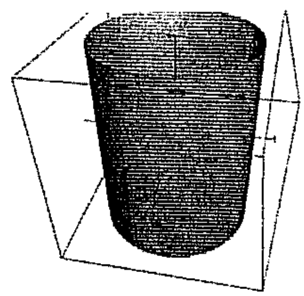
(d)

(ii)



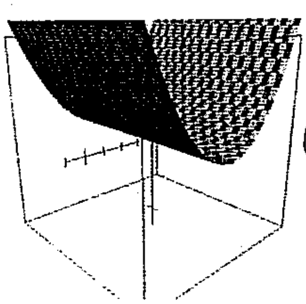
(a)

(iii)



(b)

(iv)



(c)

(v)