Test 2 Practice Key, Math 1410

Spring 2013, Dr. Graham-Squire

1. Use the limit definition of the derivative to calculate $\frac{d}{dx}(\sqrt{x-7})$.

   Ans: $\frac{d}{dx}(\sqrt{x-7}) = \lim_{h\to 0} \frac{1}{h}(\sqrt{x+h-7} - \sqrt{x-7}) = \lim_{h\to 0} \frac{1}{h} \frac{x+h-7-(x-7)}{\sqrt{x+h-7}+\sqrt{x-7}} =$
   $\lim_{h\to 0} \frac{1}{\sqrt{x+h-7}+\sqrt{x-7}} = \frac{1}{2\sqrt{x-7}}$

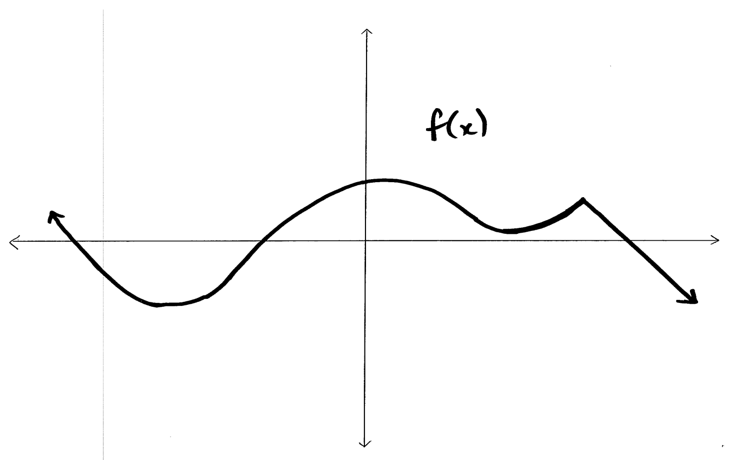
2. A particle moves along a horizontal line so that after t seconds its position is given by

$$s(t) = \frac{5}{3}t^3 - 10t^2 + 15t$$

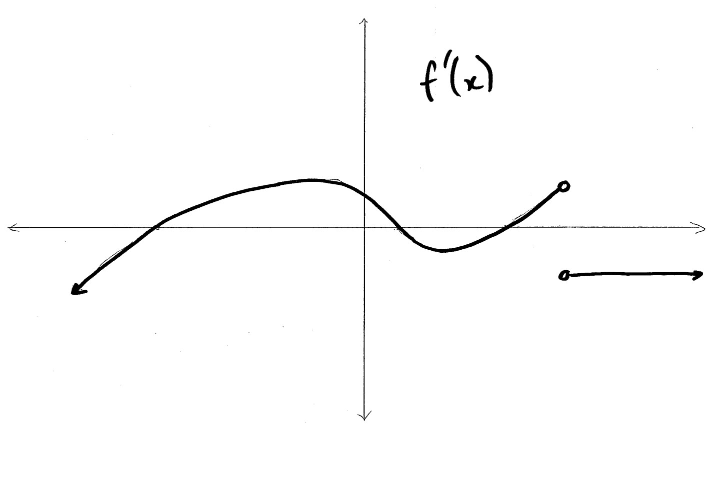
   When is the particle moving left? Note: positive direction is to the right, so increasing = moving right. (Also, you should use the derivative rules to solve this question, you do not have to use the definition of the derivative).

   Ans: The particle is moving left on the interval $(1, 3)$

3. Sketch a graph of $f'(x)$ if $f(x)$ is the graph given below:



Ans:



4. Find y' if $y = \dfrac{e^{-x}\cos x}{\ln x}$

   Ans: $\dfrac{(\ln x)(-e^{-x}\cos x - \sin x e^{-x}) - (1/x)(e^{-x}\cos x)}{(\ln x)^2}$

5. Find the x-coordinate(s) when the given function has a horizontal tangent line

$$T(x) = x^2 e^{1-3x}$$

   Ans: $x = 0$ and $x = 2/3$

6. Find $\dfrac{dy}{dx}$ if $y = \sqrt[3]{x + \sqrt{2\sec x}}$

   Ans:

$$y' = \frac{1}{3}[x + (2\sec x)^{1/2}]^{-2/3}(1 + \frac{1}{2}(2\sec x)^{-1/2} 2\sec x\tan x)$$

   or

$$y' = \frac{1}{3}[x + (2\sec x)^{1/2}]^{-2/3}(1 + \frac{1}{\sqrt{2}}(\sec x)^{1/2}\tan x)$$

7. Find y' if $\ln(xy) = e^{2x}$

   Ans: $y' = 2ye^{2x} - \dfrac{y}{x}$

8. Calculate $\dfrac{d}{dx}\tan(\arctan x)$ two different ways- First take the derivative and then simplify your answer. Next, simplify the expression first and then take the derivative. (Hint: $\cos(\arctan x) = \dfrac{1}{\sqrt{1+x^2}}$....do you know how to prove it?)

   Ans: You should get 1 both ways you do it.

9. Use logarithmic differentiation to find $\dfrac{d}{dx}(x^{\ln\sqrt{x}})$

   Ans: $\dfrac{\ln x}{x}\left(x^{\ln\sqrt{x}}\right)$

10. The volume of a right circular cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone.

(a) Find the rate of change of the volume with respect to the radius, assuming the height is held constant.

Ans: $\frac{dV}{dr} = \frac{2}{3}\pi r h$

(b) Find the rate of change of the volume with respect to the height, assuming the radius is held constant.

Ans: $\frac{dV}{dh} = \frac{1}{3}\pi r^2$

(c) Suppose $r = 3$ and $h = 4$. If you wanted to increase the volume the greatest amount, would it be more beneficial to increase the radius slightly or the height slightly? Use your results from parts (a) and (b) to explain your answer.

Ans: It would be more beneficial to increase the radius. Because when $r = 3$ and $h = 4$, the answer for (a) is 8π, which is larger than the answer for (b) (3π). That is, the volume is increasing at a faster rate as the radius changes than it is as the height changes.