Test 2A - MTH 1410

Dr. Graham-Squire, Spring 2014

Name: Key

8:02
8:13
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I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 3 questions of the test, however you should still show all of your work. No calculators are allowed on the last 5 questions of the test.
5. You will be able to come back to the calculator portion of the test, but
6. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
7. Make sure you sign the pledge.
8. Number of questions = 8. Total Points = 55.

1. (8 points) Use the limit definition of the derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

to calculate the derivative of $f(x) = \dfrac{2}{x+3}$. You can check your work by taking the derivative of f using the shortcut rules, but you will only get credit for using the limit definition.

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{x+h+3} - \frac{2}{x+3}}{h}$$

$$= \lim_{h\to 0} \frac{2(x+3) - 2(x+h+3)}{(x+h+3)(x+3)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x}+\cancel{6} - (\cancel{2x}+2h+\cancel{6})}{(x+h+3)(x+3)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{(x+h+3)(x+3)} \cdot \frac{1}{\cancel{h}}$$

goes to zero

$$= \frac{-2}{(x+3)(x+3)}$$

$$= \boxed{\frac{-2}{(x+3)^2}}$$

Compare to

$f(x) = 2(x+3)^{-1}$

$f'(x) = -2(x+3)^{-2} \cdot 1$

-1 for no $\lim_{h\to 0}$

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2. (~~8~~ points) Use the chain rule, and the fact that $\csc x = (\sin x)^{-1}$, to prove the derivative rule for $\frac{d}{dx}(\csc x)$.

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}(\sin x)^{-1}$$

$$= -1\,(\sin x)^{-2} \cdot \cos x$$

$$= \frac{-\cos x}{\sin^2 x}$$

$$= -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= -\csc x \cot x$$

-1 for quotient rule

3. (5 points) A person's IQ (I) is measured by finding their mental age M (usually by administering a certain test) and their chronological age C, then plugging those into the formula

$$I = \frac{100M}{C}.$$

both $C, M > 0$

(a) Calculate the rate of change of I with respect to M, assuming C is held constant.

(b) Calculate the rate of change of I with respect to C, assuming M is held constant.

(c) If you want to increase your IQ, is it more beneficial to increase your mental or chronological age? Explain your answer, and use your answers from (a) and (b) to support your conclusion.

(a) $\cancel{\frac{dI}{dM}} = \quad I = \frac{100}{C} \cdot M$

1.5 $\quad \Rightarrow \frac{dI}{dM} = \frac{100}{C} \cdot 1 = \frac{100}{C}$

(b) $I = (100M)\, C^{-1}$

1.5 $\quad \Rightarrow \frac{dI}{dC} = (100M)(-1)\, C^{-2} = \frac{-100M}{C^2}$

(c) Since $\frac{dI}{dM}$ is positive and $\frac{dI}{dC}$ is negative,

2 it is more beneficial to increase your mental age. That will give a positive increase in IQ, whereas an increase in C will make a decrease in IQ.

(for full points, need to mention positive vs. negative)

Test A **No Calculator** Name: Key

4. (8 points) Find y' if

$$\frac{d}{dx}\left(\ln(\tan(y)) = x^2y + 3x\right)$$

✓ ✓ ✓ ✓ ✓ 0.5 0.5

$$✓ \quad \frac{1}{\tan y} \cdot \sec^2 y \cdot y' = x^2 y' + 2xy + 3$$

$$\frac{\sec^2 y}{\tan y} y' - x^2 y' = 2xy + 3 \quad ✓$$

$$\left(\frac{\sec^2 y}{\tan y} - x^2\right) y' = 2xy + 3$$

$$\boxed{y' = \frac{2xy+3}{\left(\frac{\sec^2 y}{\tan y}\right) - x^2}} \quad ✓$$

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5. ~~(8~~ points) Find the derivative of $f(x) = \frac{4^x}{\sin x}$. (Note: if you can't remember the derivative of 4^x, set $y = 4^x$ and do logarithmic differentiation to figure it out)

$$f'(x) = \frac{\sin x \cdot 4^x \cdot \ln 4 - 4^x \cdot \cos x}{\sin^2 x}$$

$$= \frac{4^x(\sin x \ln 4 - \cos x)}{\sin^2 x}$$

$+0.5$ back if they tried to find $\frac{d}{dx}(4^x)$

(6) 8 points:

$$h(x) = (\cos(7x)) \left(\frac{1}{e^x}\right) \sqrt[5]{x}$$

$$h(x) = (\cos(7x)) \cdot e^{-x} \cdot x^{1/5}$$

$$h'(x) = \boxed{(-\sin(7x) \cdot 7)\, e^{-x} \cdot x^{1/5} + (\cos(7x))(-e^{-x})\, x^{1/5} + (\cos(7x))\, e^{-x} \cdot \frac{1}{5} x^{-4/5}}$$

or

$$\boxed{(-\sin(7x) \cdot 7)(e^{-x} x^{1/5}) + \cos(7x)\left(-e^{-x} x^{1/5} + e^{-x} \cdot \frac{1}{5} x^{-4/5}\right)}$$

~~6~~ 7. (~~5~~ 8 points) Use logarithmic differentiation to find $\frac{d}{dx}\left(x^{(x^2)}\right)$.

$$y = x^{(x^2)}$$

$$\ln y = \ln x^{x^2}$$

$$\frac{d}{dx}\left(\ln y = x^2 \cdot \ln x\right)$$

$$\frac{y'}{y} = 2x \cdot \ln x + \frac{1}{x} \cdot x^2$$

$$y' = (2x \ln x + x)\, y$$

$$\boxed{y' = (2x \ln x + x)\, x^{(x^2)}}$$

-2 if they forget $y =$ and $\frac{y'}{y}$

8. (5 points) Let $f(x) = 5\log_4 x$. Find the equation for the tangent line to f at the point (4,5). (Hint: if you can't remember the derivative for $\log_4 x$, do $y = \log_4 x$, then rewrite it into exponential form and do implicit differentiation to find y'.)

$$f'(x) = 5 \cdot \frac{1}{x(\ln 4)}$$

$$f'(4) = 5 \cdot \frac{1}{4(\ln 4)} = \frac{5}{4(\ln 4)}$$

$$y - y_1 = m(x - x_1)$$

$$\uparrow \quad \uparrow \quad \uparrow$$

$$5 \quad \frac{5}{4(\ln 4)} \quad 4$$

$$\boxed{y - 5 = \frac{5}{4(\ln 4)}(x - 4)}$$

$$y = \frac{5}{4(\ln 4)}(x-4) + 5$$

+0.5 for trying to figure out

9 ~~8~~. (8 points) Find the derivative of

$$g(x) = e^{\arcsin x}\sqrt{1-x^2}.$$

Simplify your answer by canceling and/or factoring out common terms.

$$g'(x) = e^{\arcsin x} \cdot \frac{1}{\cancel{\sqrt{1-x^2}}} \cdot \cancel{\sqrt{1-x^2}} + e^{\arcsin x} \cdot \frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-\cancel{2}x)$$

$$= e^{\arcsin x} + e^{\arcsin x}\left(\frac{-x}{\sqrt{1-x^2}}\right)$$

$$= \boxed{e^{\arcsin x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right)}$$

or

$$e^{\sin^{-1} x}\left(\frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}}\right)$$

Extra Credit(1.5 points) Calculate the derivative of $\tan\sqrt{\ln(7)}$.

$\boxed{0}$ because $\tan\sqrt{\ln 7}$ is a constant.