Minitest 1A - MTH 1410

Dr. Graham-Squire, Spring 2014

3:30

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 3 questions of the test, however you should still show all of your work. No calculators are allowed on the last 2 questions of the test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 5. Total Points = 30.

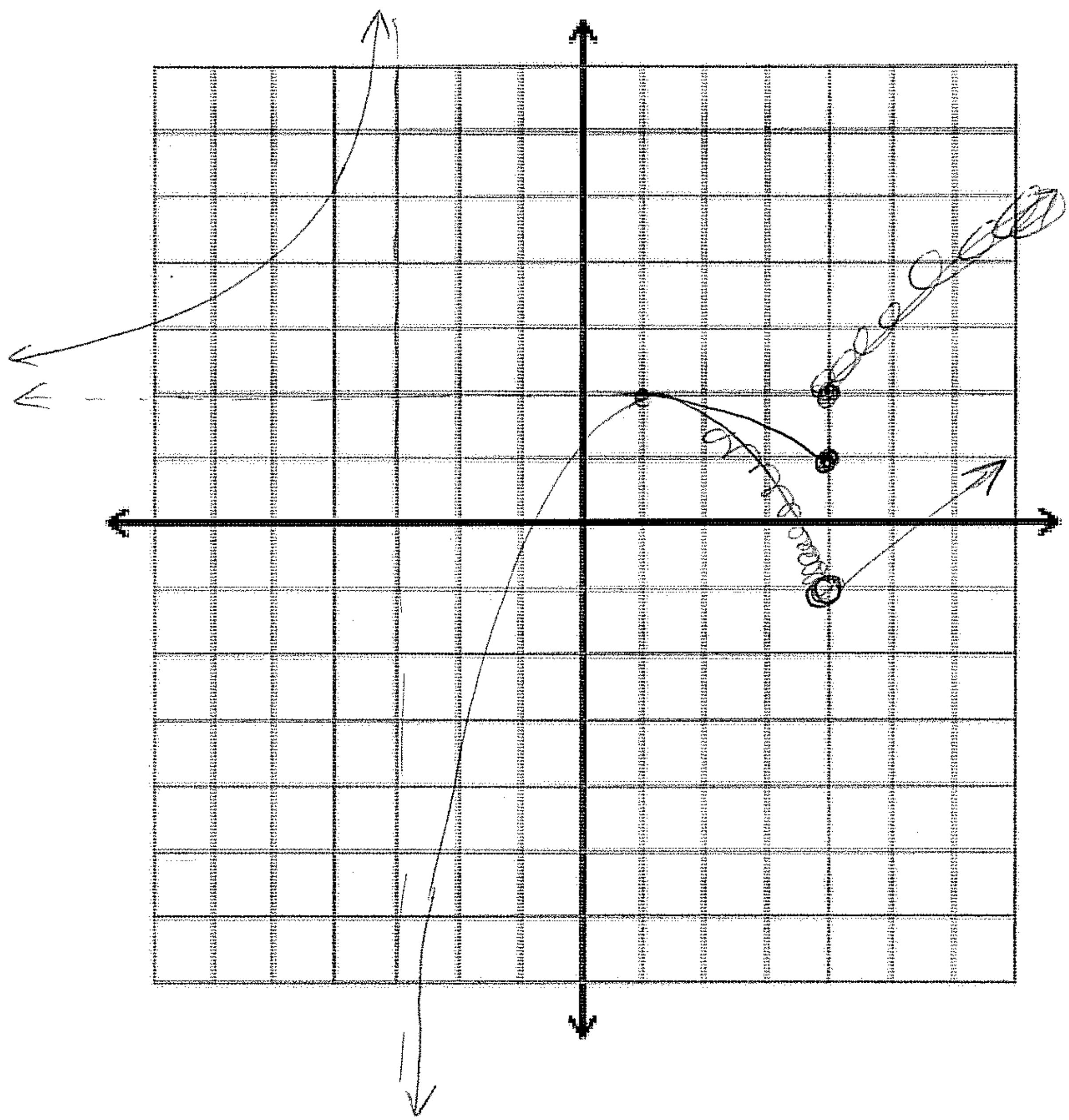
1. (6 points) On the graph below, sketch a function $f(x)$ that satisfies the following properties. Note that there is more than one correct answer!

$\lim_{x\to(-\infty)} f(x) = 2$ $\qquad$ $f(1) = 2$

$\lim_{x\to(-3)^-} f(x) = \infty$ $\qquad$ $f'(1) = 0$

$\lim_{x\to(-3)^+} f(x) = -\infty$

$\lim_{x\to 4} f(x) = -1$ and $f(x)$ has a discontinuity at $x = 4$



2. (6 points) Use the <u>definition of the derivative</u> to calculate $f'(1)$ for $f(x) = \frac{1}{\sqrt{x}}$

$$f'(4) = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h}$$

$$= \lim_{h\to 0} \left(\frac{3}{\sqrt{4+h}} - \frac{3}{\sqrt{4}}\right)\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \left(\frac{2\cdot 3 - 3\sqrt{4+h}}{2\sqrt{4+h}}\right)\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(6-3\sqrt{4+h})(6+3\sqrt{4+h})}{2\sqrt{4+h}\,(6+3\sqrt{4+h})}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{36-9(4+h)}{2\sqrt{4+h}\,(6+3\sqrt{4+h})}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{36-36-9h}{}$$

$$f(1+h)-f(1)$$

$$\left(\frac{1}{\sqrt{1+h}} - \frac{1}{1}\right)\frac{1}{h}$$

$$\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\cdot\frac{1}{h}$$

$$\frac{(1-(1+h))}{(\sqrt{1+h})(1+\sqrt{1+h})} - 1$$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\left(\frac{1}{\sqrt{1+h}} - \frac{1}{\sqrt{1}}\right)\frac{1}{h}$$

$$= \lim_{h\to 0}\left(\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\right)\frac{1}{h}$$

$$= \lim_{h\to 0}\left(\frac{(1-\sqrt{1+h})\cdot(1+\sqrt{1+h})}{\sqrt{1+h}\,(1+\sqrt{1+h})}\right)\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{1-(1+h)}{\sqrt{1+h}\,(1+\sqrt{1+h})}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}\,-1}{\sqrt{1+h}\,(1+\sqrt{1+h})}\cdot\frac{1}{\cancel{h}} = \frac{-1}{\sqrt{1}\,(1+\sqrt{1})} = \boxed{\frac{-1}{2}}$$

3. (6 points) For what value of c will the function be continuous for all real numbers? Make sure to show your work and use the definition of continuity as part of your explanation.

$$f(x) = \begin{cases} 2c & \text{if } x \le 0 \\ \dfrac{1}{x}\left(\dfrac{5}{2} - \dfrac{5}{2-x}\right) & \text{if } x > 0 \end{cases}$$

Need $\lim_{x \to 0} f(x) = f(0) = 2c$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{1}{x}\left(\frac{5}{2} - \frac{5}{2-x}\right)$$

$$= \lim_{x \to 0^+} \frac{1}{x}\left(\frac{5(2-x) - 5(2)}{2(2-x)}\right)$$

$$= \lim_{x \to 0^+} \frac{1}{\cancel{x}}\left(\frac{\cancel{10} - 5\cancel{x} - \cancel{10}}{2(2-x)}\right)$$

$$= \frac{-5}{4}$$

$$\Rightarrow \frac{-5}{4} = 2c \quad \Rightarrow \boxed{c = \frac{-5}{8}}$$

Test A **No Calculator** Name: Key

Fix

4. (6 points) Calculate the limits. Make sure to show your work and use correct notation!

(a) $\lim_{x\to 3^-} \frac{x^2 - 3x - 4}{x^2 - \cancel{2}^{4}x + 3} = \lim_{x\to 3^-} \frac{(x-4)(x+1)}{(x-3)(x-1)} \longrightarrow = \frac{(-1)(4)}{0(2)} = \frac{-4}{0}$

$\Rightarrow -\infty$ or ∞

As $x \to 3^-$, $x < 3 \Rightarrow (x-3) < 0$

and $(x-1) > 0$

$\Rightarrow \frac{-4}{-} \Rightarrow \frac{-}{-} = \boxed{+\infty}$

(b) $\lim_{x\to(-2)^+} \frac{x^2 - x - 6}{x^2 + 2x} = \lim_{x\to(-2)^+} \frac{(x-3)\cancel{(x+2)}}{x\cancel{(x+2)}} \longrightarrow \frac{0}{0}$

$= \lim_{x\to(-2)^+} \frac{x-3}{x} = \frac{-2-3}{-2} = \frac{-5}{-2} = \boxed{\frac{5}{2}}$

5. (6 points) Calculate each limit. Explain your reasoning or show it in a mathematically correct way. If the limit does not exist, explain (briefly) why.

(a) $\lim_{x\to(-\infty)} \frac{3x^6-2x}{4x^3-x^5} =$

$$\lim_{x\to(-\infty)} \frac{(3x^6-2x)\frac{1}{x^5}}{(4x^3-x^5)\frac{1}{x^5}}$$

$$= \lim_{x\to(-\infty)} \frac{3x-\frac{2}{x^4}}{\frac{4}{x^2}-\frac{1}{1}} = \frac{3(-\infty)}{0-1} = \frac{-\infty}{-1} = \boxed{\infty}$$

$\frac{2}{x^4} \to 0$, $\frac{4}{x^2} \to 0$

(b) $\lim_{x\to(-\infty)} \frac{10}{2+3e^x} =$

$$e^{-\infty} = \frac{1}{e^{\infty}} = 0 \Rightarrow \lim_{x\to(-\infty)} 2+3e^x = 2+3(0)=2$$

$$\Rightarrow \lim_{x\to(-\infty)} \frac{10}{2+3e^x} = \frac{10}{2} = \boxed{5}$$

Extra Credit(1 point) Calculate $\lim_{x\to\infty} \frac{\sin x}{x}$.

as $x\to\infty$ $-1 \le \sin x \le 1$

but $x \to \infty$

$\Rightarrow$ like $\frac{\pm 1}{\infty} = 0$