MTH 1410, SPRING 2014
DR. GRAHAM-SQUIRE

OPTIMIZATION PROJECT - SECTION 4.6

An oil company wants to build a pipeline to take oil from an oil well to a refinery. Unfortunately, the well and the refinery are on either side of a straight river which is 10 miles wide, and they are 50 miles apart along the coastline (that is, if you want to go from the well to the refinery you must first cross 10 miles of river and then go 50 miles along the side of the river). The company has hired you to figure out the cheapest way to build the pipeline, and you need to clearly explain your solution so that the less mathematically-sophisticated oil people will understand.

It costs $200 per mile to lay pipe across the river but only $160 per mile to lay the pipe over land. There is also one other potential cost: It costs extra money each time that you have a bend in the pipeline. If you go straight across the river and use an L-shaped bend it costs an extra $150. If you lay the pipe diagonally across the river but hit land before you get to the refinery, you have to use a slanted L-shaped bend (like an obtuse angle). These have to be custom made and they cost $1000. If you go directly diagonally across the river to the refinery without touching any land, then you do not have to pay extra for a bend (since there will not be one).

Exercise 1. Find the cheapest path to lay the pipeline by doing the following:

(a) Draw a diagram of the situation. Include any variables that you are going to use in the rest of your answer.

(b) Calculate the cheapest way to build the pipeline for the given situation.

(i) You will need to look at 3 different scenarios, and for one of them you will need to use calculus to minimize the cost of laying the pipeline.

(ii) Use either the first or second derivative test to prove that your result is a minimum.

(c) What is the effect of the extra cost for a bend in the pipeline? If there was no extra cost for the bend, would you have a different answer for what the cheapest path would be?

Exercise 2. Suppose the situation was modified as follows. Instead of a river 10 miles wide, suppose the first 8 miles across from the oil well are water and the last 2 mile width is covered by a swamp. It costs $250 per mile to lay pipe in the swamp, and there is no extra cost to put a bend in the pipeline. To find the optimal cost in this situation, you would need to use multivariable calculus, which is Calculus III, so I do not expect that. Instead, answer the following questions. I expect these to be thorough answers- if you think just a sentence or two will suffice, you will probably NOT get the full amount of points.

(i) Use your intuition from what you found in Exercise 1 to make a guess at what you think the cheapest path would be. Explain why you think it would be the cheapest path. You should reference your answer from Exercise 1 to bolster your explanation. Calculate the cost for that path.

(ii) Use your intuition to come up with two other paths that you think could be cheaper (again, explain why you think they could be cheaper), then calculate the cost of the two other paths of your choosing to compare to your first guess. Was your first guess the cheapest way to go? If not, explain why you think you were wrong.

Timeline:

(a) Exercise 1 is due Monday, March 24, before 4 pm. You can either hand it in to me in class on that day or drop it by my office before 4. If I am not in my office you can slide it under the door. This will account for 60% of the grade for this project.

(b) The complete project is due Wednesday, April 2, at the beginning of class. 10% of your project grade will be corrections to Exercise 1 and the remaining 30% will be Exercise 2.