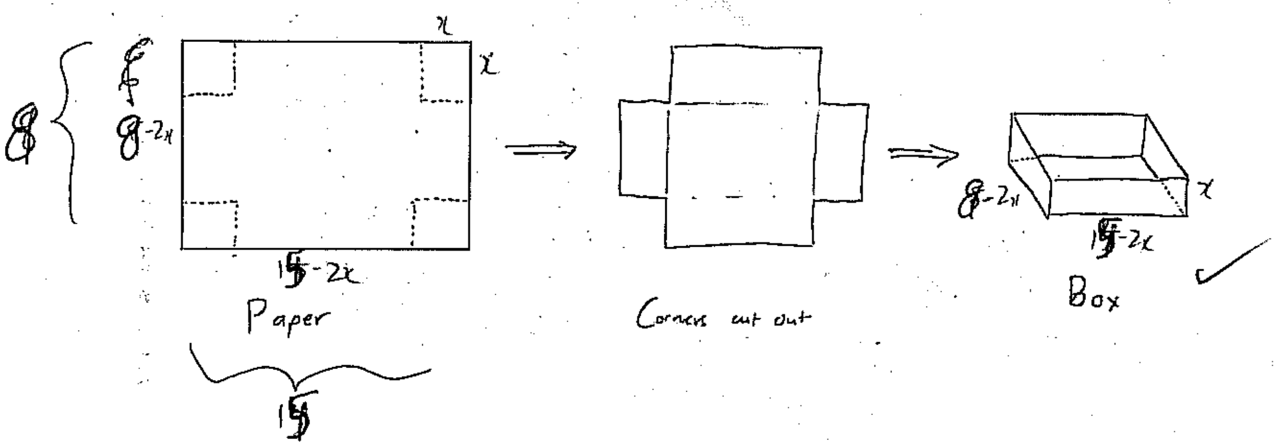
2:49
2:57
8 min
⇒ give 25 min. in class.

Quiz 5B, Calculus I

Dr. Graham-Squire, Spring 2013

Name: Key

1. (4 points) A rectangular piece of paper of size ~~14×9~~ 15×8 inches is given. You want to cut out identical squares from each corner and then fold up the edges to make an open-topped box of *maximum* volume (see diagram). Use calculus to find out what will be the dimensions of the box that has the maximum volume. Make sure to confirm that your solution is a maximum.



$$\text{Volume} = x(8-2x)(15-2x)$$

$$V(x) = x(120 - 30x - 16x + 4x^2)$$

$$V(x) = 120x - 46x^2 + 4x^3$$

$$V'(x) = 120 - 92x + 12x^2$$

$$0 = 120 - 92x + 12x^2$$

$$x = \frac{92 \pm \sqrt{92^2 - 4(120)(12)}}{24}$$

$$= \frac{92 \pm 52}{24} = \frac{40}{24} \text{ or } 6$$

6 is too long ⇒ $x = \frac{5}{3} = 1.67$

$+$ | $-$ at 1.67 ⇒ Max

dimensions are

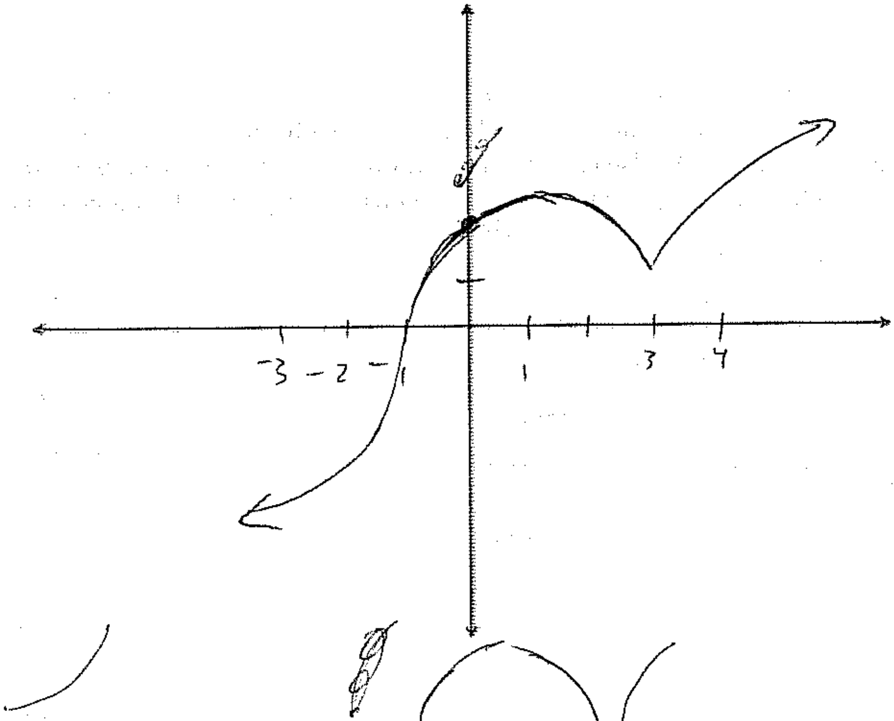
length $= 15 - \frac{10}{3} = \frac{35}{3}$

width $= 8 - \frac{10}{3} = \frac{14}{3}$

height $= \frac{5}{3}$

2. (3 points) Sketch the graph of a continuous function $f(x)$ that satisfies the following:

- $f(0) = 2$
- $f'(x) > 0$ on intervals $(-\infty, 1)$ and $(3, \infty)$
- $f'(x) < 0$ on $(1, 3)$.
- $f''(x) > 0$ on the intervals $(-\infty, -1)$
- $f''(x) < 0$ on $(-1, 3)$ and $(3, \infty)$.



3. (3 points) Use calculus to find the absolute maximum and absolute minimum of the function $f(x) = 2x^3 - \frac{5}{2}x^2 - 1$ on the interval [-1,1].

$f'(x) = 6x^2 - 5x = x(6x-5)$ ✓

$0 = x(6x-5)$ at $x = 0$, $0 = 6x - 5$
✓
$\frac{5}{6} = x$ ✓

Check $f(-1) = -5.5$ → Abs. min

$f(0) = -1$ → Abs. Max

$f(\frac{5}{6}) = -1.57$

$f(1) = -1.5$

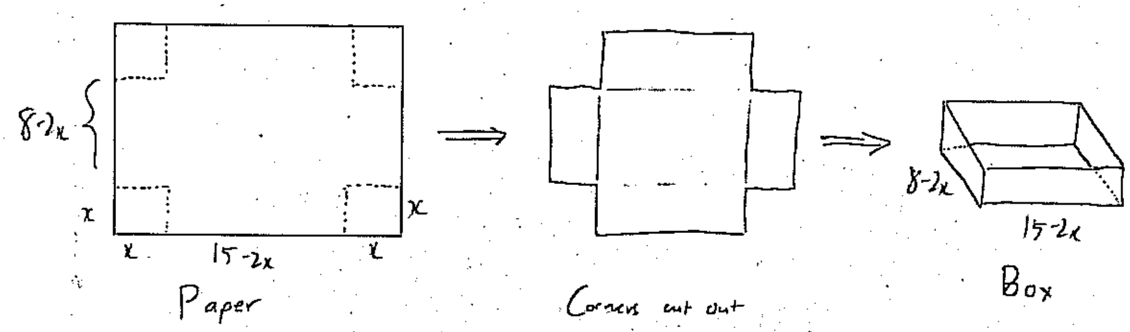
✓✓

Quiz 5A, Calculus I

Dr. Graham-Squire, Spring 2013

Name: Key

1. (4 points) A rectangular piece of paper of size 15×8 inches is given. You want to cut out identical squares from each corner and then fold up the edges to make an open-topped box of *maximum* volume (see diagram). Use calculus to find out what will be the dimensions of the box that has the maximum volume. Make sure to confirm that your solution is a maximum.



$\text{Volume} = x(8-2x)(15-2x)$

$V(x) = x(120-46x+4x^2)$

$V(x) = 120x - 46x^2 + 4x^3$

$V'(x) = 120 - 92x + 12x^2$

$\dfrac{0}{4} = \dfrac{120-92x+12x^2}{4}$

$0 = 30 - 23x + 3x^2$

$0 = (3x-5)(x-6)$

$\Rightarrow 3x-5=0$ or $x=6$ → too big, x can be at most 4.

$x = 5/3$

dimensions: height $= 5/3 = 1.67$

length $= 15-2(\frac{5}{3}) = 11.67$

width $= 8-2(\frac{5}{3}) = 4.67$

$f'(1) = +$

$f'(2) = -$

$+ \quad 5/3 \quad -$

→ max

2. (3 points) Use calculus to find the absolute maximum and absolute minimum of the function $f(x) = x^3 - \frac{7}{2}x^2 + 3$ on the interval [-1,4].

$f'(x) = 3x^2 - 7x$

$0 = x(3x-7)$

$\Rightarrow x = 0$ or $3x - 7 = 0$

$x = 7/3$

check: $f(-1) = -1.5$

$f(0) = 3$

$f(7/3) = -3.35$ abs. min

$f(4) = 64 - 56 + 3 = 11$ abs. max

3. (3 points) Sketch the graph of a continuous function $f(x)$ that satisfies the following:
- $f(0) = 0$
- $f'(x) > 0$ on intervals $(-\infty, -1)$ and $(1, \infty)$
- $f'(x) < 0$ on $(-1, 1)$.
- $f''(x) > 0$ on the intervals $(-\infty, -1)$ and $(-1, 3)$
- $f''(x) < 0$ on $(3, \infty)$.