Test 2 - MTH 1410

Dr. Adam Graham-Squire, Fall 2017

12.5 min.

→ 65 minutes Group

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(signature)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 2 questions of the test, however you should still show all of your work. No calculators are allowed on the last 7 questions of the test.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge above.
7. Number of questions = 9. Total Points = 45.

1. (6 points) ~~(a)~~ Find the tangent line to the graph of

$$\frac{y^2+1}{3y-1} = x$$

at the point (1,1). You should write your final answer as a line in the form of $y = mx+b$ or $y - y_1 = m(x - x_1)$.

$$\frac{d}{dx}\left(\frac{y^2+1}{3y-1}\right) = \frac{d}{dx}(x)$$

$$\frac{(3y-1)(2yy') - (y^2+1)(3y')}{(3y-1)^2} = 1$$

idea

$$y'\left[(3y-1)(2y) - (y^2+1)3\right] = (3y-1)^2$$

$$y' = \frac{(3y-1)^2}{(3y-1)(2y)-(y^2+1)3}$$

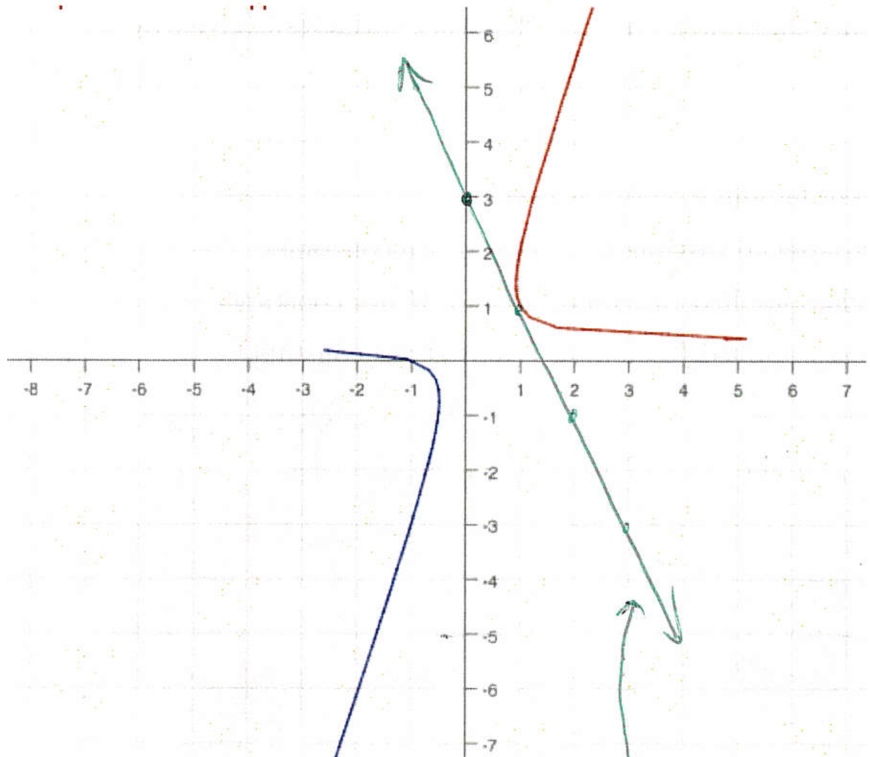
let $y=1$ $\quad y' = \frac{(2)^2}{(2)(2)-2(3)} = \frac{4}{-2} = -2$

$$\Rightarrow \quad y - y_1 = m(x-x_1)$$

$$y - 1 = -2(x-1)$$

$$\boxed{y = -2x+3}$$

2. (2 points) Below is the graph of $\dfrac{y^2+1}{3y-1} = x$. Sketch the tangent line you found in question 1 to verify that it is in fact tangent at the point (1,1). (Note: If you cannot find the answer for (a), or your answer for (a) seems wrong, you should sketch in by hand what you think the tangent line at (1,1) would look like).



$\boxed{y=-2x+3}$

No Calculator Name: Key

3. (6 points) Eva's happiness depends on the amount of Lip gloss (L), Mascara (M), and Blush (B) that she can apply to her face, without her parents finding out. The relationship between Eva's happiness (H) and the other variables is given by

$$H = \frac{B^3(\sin B)\sqrt{L}}{e^M}$$

Find the rate of change of happiness with respect to

(a) the amount of Blush applied (assuming L and M are constant)

(b) the amount of Lip gloss applied (assuming B and M are constant)

(a) the amount of Mascara applied (assuming L and B are constant)

✓ for idea

(a) $\boxed{\frac{dH}{dB} = \frac{\sqrt{L}}{e^M} \cdot \left(3B^2(\sin B) + (\cos B) B^3\right)}$ 2

(b) $\boxed{\frac{dL}{dB} = \frac{B^3(\sin B)}{e^M} \cdot \frac{1}{2} L^{-1/2}}$ 1.5

(c) $H = B^3(\sin B)\sqrt{L} \cdot e^{-M}$ 1.5

$$\boxed{\frac{dH}{dM} = B^3(\sin B)\sqrt{L} \cdot (-e^{-M})}$$

4. (6 points) Use logarithmic differentiation to calculate the derivative of

$$f(x) = x^{\arcsin(x)}$$

$$y = x^{\arcsin(x)}$$ 0.5

$$\ln y = \ln x^{\arcsin x}$$

$$\frac{d}{dx}\left(\ln y = (\arcsin x)(\ln x) \right)$$

0.5 $\qquad$ 0.5 $\qquad$ 0.5

$$\frac{y'}{y} = \frac{1}{\sqrt{1-x^2}} \cdot \ln(x) + (\arcsin x) \cdot \frac{1}{x}$$

$$\boxed{y' = \left(\frac{\ln x}{\sqrt{1-x^2}} + \frac{\arcsin x}{x} \right) x^{\arcsin x}}$$

-1 if no product rule

-0.5 if y instead of $x^{\arcsin}$

5. (5 points) Calculate the derivative of $h(x)$ below. Simplify your answer as much as possible.

$$h(x) = x(x^2)(\sqrt{x})x^{2/3}$$

3

$$h(x) = x^1 \cdot x^2 \cdot x^{1/2} \cdot x^{2/3}$$

$$h(x) = x^{1+2+\frac{1}{2}+\frac{2}{3}}$$

$$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$$

2

$$h(x) = x^{\frac{25}{6}}$$

$$h'(x) = \boxed{\frac{25}{6} x^{19/6}}$$

or

$$h'(x) = 1 \cdot x^2 (x^{1/2}) x^{2/3} + x(2x)x^{1/2}x^{2/3} + x(x^2)\left(\frac{1}{2}x^{-1/2}\right)x^{2/3} + x(x^2)x^{1/2} \cdot \frac{2}{3}x^{-1/3}$$

3

$$= x^{19/6} + 2x^{19/6} + \frac{1}{2}x^{19/6} + \frac{2}{3}x^{19/6}$$

2

$$= \frac{25}{6}x^{19/6}$$

$W\ R_2 R_3 L$

no use

Possibilities: $R_3 \hookrightarrow L$ → L wins
→ R_2 wins ✗ can't happen

only chance is $R_3 \hookrightarrow L$, then R_3 gives to L, surpassing R_2, L wins vs. W

⇒

any order
✓ $R_3\ L\ R_2$
✓ $R_3\ L\ W$
✓ $R_3\ L$
1st ✓ $R_3\ R_2\ L$

$R_2 \hookrightarrow L$ ⇒ L wins

⇓

R_3 wins

✓ $R_2\ L\ R_3$
✓ $R_2\ R_3$
✓ $R_2\ R_3\ L$
✓ $R_2\ R_3\ W$

✓ $R_2\ R_3\ L$
✓ $R_2\ L$
✓ $R_2\ L\ R_3$
✓ $R_2\ L\ W$

```
tic void
  monoSwapCheck (~~ArrayList<String>~~ perms, ~~ArrayList<Integer> votes~~, ~~int~~ gap,
     int checkPos, int swapPos)
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#line zoomin

Thomasville Teacher Training
Day 1 Evaluation

1) What part of the training did you enjoy the most and/or find the most useful?

different bases →
never seen before 3 challenging
- back in student mode

2) What part of the training did you enjoy the least and/or find the least useful?

fractions b/c I get those pretty easily

diff. bases b/c I can't apply the math part

3) Do you have any suggestions for things to cover in the future (this can be specific math content, teaching techniques, activities, etc), or things to avoid?

♥ed it!
Thanks for challenging me!

6. (5 points) Calculate $f'(x)$ and $f''(x)$ for

$$f(x) = e^{x^3 - x^2}$$

You do not need to simplify your answers, though you can if you think it would help you.

2 $f'(x) = e^{x^3-x^2} \cdot (3x^2-2x)$

3 $\boxed{f''(x) = e^{x^3-x^2}(3x^2-2x)\cdot(3x^2-2x) + e^{x^3-x^2}\cdot(6x-2)}$

-0.5 if no ( )

-1.5 if no product rule for f''

7. (5 points) Calculate the derivative of the function given below. You do NOT need to simplify your answer.

$$m(x) = \left(\frac{\csc x}{4^x \cdot \ln(x^2)}\right) \cdot x^4$$

$$m(x) = \frac{(\csc x)\, x^4}{4^x \cdot (2\ln x)}$$

$$m'(x) = \frac{4^x (2\ln x)\left\{\left[(-\csc x \cot x)x^4 + 4x^3(\csc x)\right]\right\} - (\csc x)\cdot x^4 \cdot \left[4^x (\ln 4)\cdot 2\ln x + 4^x \cdot 2 \cdot \tfrac{1}{x}\right]}{\left[4^x \cdot (2\ln x)\right]^2}$$

or

$$m'(x) = \frac{(4^x \cdot \ln(x^2)) \cdot (-\csc x \cot x) - \csc x\left(4^x \cdot (\ln 4)\cdot \ln(x^2) + 4^x \cdot \frac{1}{x^2} \cdot 2x\right)}{\left[4^x \cdot \ln(x^2)\right]^2} \cdot x^4 + 4x^3\left(\frac{\csc x}{4^x \cdot \ln(x^2)}\right)$$

8. (5 points) Use the quotient rule (or the chain and product rules) to prove the derivative rule for $y = \cot(x)$.

$$y = \cot x$$

$$y = \frac{\cos x}{\sin x}$$

$$y' = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$$

$$y' = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$\boxed{y' = -\csc^2 x}$$

9. (5 points) Calculate the derivative of the function given below. You do NOT need to simplify your answer.

$$y = \sqrt[3]{6x^5 - \tan(\sqrt[3]{4x-7})}$$

$$y = \left(6x^5 - \tan(4x-7)^{1/3}\right)^{1/3}$$

$$y' = \frac{1}{3}\left(6x^5 - \tan(4x-7)^{1/3}\right)^{-2/3} \cdot \left[30x^4 - \sec^2(4x-7)^{1/3} \cdot \frac{1}{3}(4x-7)^{-2/3} \cdot 4\right]$$

Extra Credit(2 points) Use logarithmic differentiation to prove the quotient rule.

$$h(x) = \frac{f(x)}{g(x)}$$

$$\Rightarrow \quad y = \frac{f(x)}{g(x)}$$

$$\ln y = \ln\left(\frac{f(x)}{g(x)}\right)$$

$$\frac{d}{dx}\left(\ln y = \ln f(x) - \ln g(x)\right)$$

$$\frac{y'}{y} = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}$$

$$y' = \left(\frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}\right)\frac{f(x)}{g(x)}$$

$$y' = \frac{f'(x)}{\cancel{f(x)}} \cdot \frac{\cancel{f(x)}}{g(x)} - \frac{g'(x)f(x)}{[g(x)]^2}$$

$$y' = \frac{f'(x)\cdot g(x)}{[g(x)]^2} - \frac{g'(x)f(x)}{[g(x)]^2}$$

$$\boxed{y' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}}$$