Quiz 1A, Calculus I - Calculators Okay

Dr. Graham-Squire, Fall 2017

5 min ⇒ 20

Name: Key

1. (5 points) At what two x-value(s) is $f(x)$ discontinuous? For each point of discontinuity, explain what part of the definition of continuity fails at that point, and how the function fails it. A graph might help you, but it is NOT enough to just reference the graph.

$$f(x) = \begin{cases} \dfrac{x^2+8x+15}{x+3} & \text{if } x \le 2 \\ \sqrt{x+7} & \text{if } x > 2 \end{cases}$$

- Could have issue at $x=-3$, b/c makes zero in denominator

Since $f(-3) = \dfrac{(-3)^2+8(-3)+15}{(-3)+3} = \dfrac{0}{0}$ is undefined, $\boxed{x=-3}$ is a discontinuity.

- Other issue could be at $x=2$ b/c of split function.

Need $\lim\limits_{x\to 2} f(x) = f(2)$

$f(2) = \dfrac{2^2+8(2)+15}{2+3} = \dfrac{35}{5} = 7$

$\lim\limits_{x\to 2^-} f(x) =$ same $= 7$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \sqrt{x+7} = \sqrt{2+7} = \sqrt{9} = 3$

Since $7 \ne 3$, $\lim\limits_{x\to 2} f(x)$ DNE ⇒ discontinuity at $\boxed{x=2}$

2. (5 points) Calculate the limits.

$$\frac{\sqrt{4-3}-1}{4-4} = \frac{0}{0} \quad \text{Rat!}$$

(a) $\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4}$. You should be able to solve this one without using a calculator. Make sure to show your work and use correct limit notation!

$$\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4} \cdot \frac{\sqrt{x-3}+1}{\sqrt{x-3}+1}$$

$$= \lim_{x\to 4} \frac{x-3-1}{(x-4)(\sqrt{x-3}+1)}$$

$$= \lim_{x\to 4} \frac{\cancel{x-4} \quad 1}{\cancel{(x-4)}(\sqrt{x-3}+1)}$$

$$= \frac{1}{\sqrt{4-3}+1} = \boxed{\frac{1}{2}}$$

(b) $\lim_{x\to\infty} \frac{5x^4-3x+7}{2x^4+9x^2}$. You can solve this one with or without a calculator. In either case, show your work and/or explain your reasoning.

$$= \lim_{x\to\infty} \frac{(5x^4-3x+7)\frac{1}{x^4}}{(2x^4+9x^2)\frac{1}{x^4}}$$

$$= \lim_{x\to\infty} \frac{\frac{5x^4}{x^4} - \frac{3x}{x^4} + \frac{7}{x^4}}{\frac{2x^4}{x^4} + \frac{9x^2}{x^4}}$$

$$= \lim_{x\to\infty} \frac{5 - \frac{3}{x^3} + \frac{7}{x^4}}{2 + \frac{9}{x^2}}$$

$$= \frac{5-0+0}{2+0}$$

$$= \boxed{\frac{5}{2}}$$

Quiz 1B, Calculus I - Calculators Okay

Dr. Graham-Squire, Fall 2017

Name: Key

1. (5 points) Calculate the limits.

   (a) $\lim_{x\to\infty} \frac{8x^3-2x+1}{2x^3+x-13}$. You can solve this one with or without a calculator. In either case, show your work and/or explain your reasoning.

$$= \lim_{x\to\infty} \frac{(8x^3-2x+1)\cdot\left(\frac{1}{x^3}\right)}{(2x^3+x-13)\cdot\left(\frac{1}{x^3}\right)}$$

$$= \lim_{x\to\infty} \frac{8-\frac{2}{x^2}+\frac{1}{x^3}}{2+\frac{1}{x^2}-\frac{13}{x^3}}$$

$$= \frac{8-0+0}{2+0-0} = \frac{8}{2} = \boxed{4}$$

   (b) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$. You should be able to solve this one without using a calculator. Make sure to show your work and use correct limit notation!

$$= \lim_{x\to 3} \frac{(\sqrt{x+1}-2)}{(x-3)}\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}\,1}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$= \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

2. (5 points) At what two x-value(s) is $f(x)$ discontinuous? For each point of discontinuity, explain what part of the definition of continuity fails at that point, and how the function fails it. A graph might help you, but it is NOT enough to just reference the graph.

$$f(x) = \begin{cases} \dfrac{x^2+8x+12}{x+2} & \text{if } x \leq 3 \\ \sqrt{x+13} & \text{if } x > 3 \end{cases}$$

Discontinuity issues can arise if you divide by zero, so denominator of $x+2$ could be an issue, also when the function "splits" at $x=3$.

- At $x=-2$, get $f(-2) = \dfrac{(-2)^2+8(-2)+12}{-2+2} = \dfrac{0}{0}$ is undefined, so $f(-2)$ dne $\Rightarrow$ $\boxed{\text{Not continuous at } x=-2}$

  b/c need $f(-2) = \lim_{x \to (-2)} f(x)$, and $f(-2)$ dne.

- At $x=3$, Need $\lim_{x \to 3} f(x) = f(3)$

$f(3) = \sqrt{3+13} = \sqrt{16} = 4$, which is same as $\lim_{x \to 3^+} f(x)$

but, $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \dfrac{x^2+8x+12}{x+2}$

$= \dfrac{3^2+8(3)+12}{3+2} = \dfrac{45}{5} = 9$

So $\lim_{x \to 3} f(x)$ DNE, b/c $9 \neq 4$, thus $f(x)$ is $\boxed{\text{discontinuous at } x=3}$