

Test 3 Review Answers

1. Dominic has attached his new baby brother/sister to a kite and is letting the baby drift away in the wind. Assuming that the kite stays at a constant height of 100 feet above the ground and kite string is coming out of the spool at a constant rate of $5\sqrt{3}$ feet/minute, find the rate at which the angle of elevation (that is, the angle between the kite string and the ground) is changing when the kite string is 200 feet long.

Ans: $-1/40$ radians/min

2. Calculate $\lim_{x \rightarrow \pi^+} \frac{x - \pi}{\cos x}$ and $\lim_{x \rightarrow \infty} \ln(x^4 + 3) \cdot x^{-2}$

Ans: 0 and 0. For the first one you cannot apply L'Hospital's rule.

3. Sketch the graph of $f(x)$ given that:

- $f(0) = 3$ and $f(1) = 0$
- $x = 2$ is a vertical asymptote.
- $f'(x) > 0$ on $(-\infty, -1)$, $(2, 5)$ and $(5, \infty)$
- $f'(x) < 0$ on $(-1, 2)$.
- $f''(x) > 0$ on $(-\infty, -2)$ and $(5, \infty)$
- $f''(x) < 0$ on $(-2, 2)$ and $(2, 5)$
- $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow (-\infty)} f(x) = 1$

Ans: I will scan in an answer when I get a chance. It should have an inflection point at $x=-2$, local max at $x=-1$, vertical asymptote at $x=2$ with either side going to negative infinity, and an inflection point at $x=5$ where the curve has a horizontal tangent line (but it is not a local max or min at $x=5$).

4. A farmer wants to use fencing to construct a rectangular pen and subdivide it into six equal rectangles. Thus the fencing will be used for both the perimeter of the pen and the pieces that go across the inside to form the subdivisions. The farmer has exactly 408 meters of fencing. Find the outer dimensions of the pen he can build with the maximum possible area. Make sure that your answer is a maximum.

Ans: 68 by 51 meters.

5. A ball is dropped from a building 100 feet high. Given that acceleration due to gravity is -32 feet/second², find the velocity of the ball when it hits the ground.

Ans: -80 feet/sec

6. Use Newton's Method to approximate $\sqrt[5]{1000}$ correct to four decimal places.

Ans: Check your answer on your own by calculating $\sqrt[5]{1000}$ on your calculator.

7. Find the most general antiderivative of $f'(x) = x(x+2)^2 + \sec^2 x$.

Ans: $\frac{1}{4}x^4 + \frac{4}{3}x^3 + 2x^2 - \tan x + C$

8. Find the absolute maximum and absolute minimum of the function $f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 9x + 2$ on the interval $[-2, 5]$.

Ans: max of $9.875 = f(-3/2)$ and min of $-20.5 = f(3)$. Assuming I did my calculations correctly.