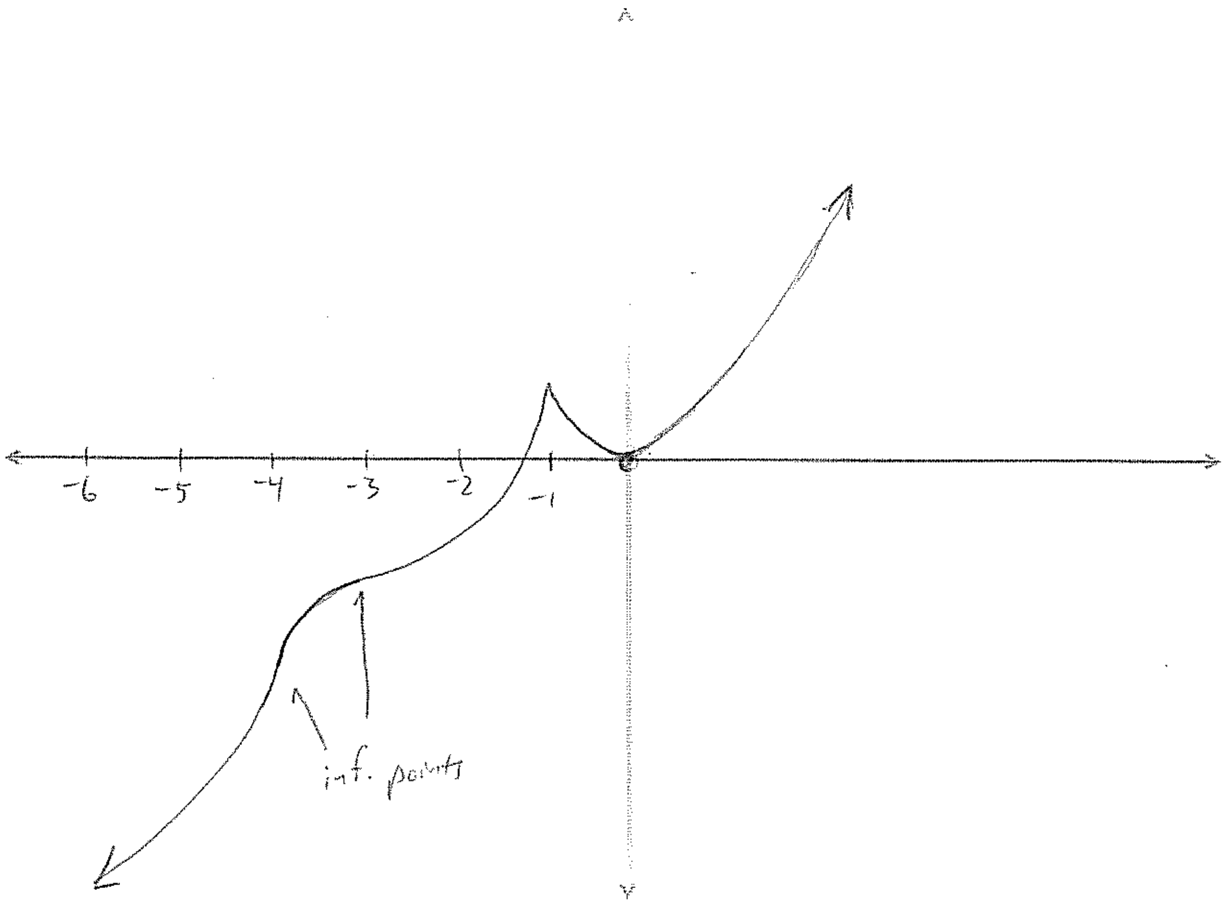
Quiz 5A - MTH 1410

12:36
12:45
9
⇒ Give 20 minutes.

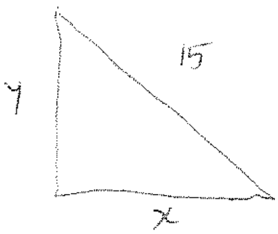
Name: Key

1) (3 points) Graph a *continuous* function f with the following properties:

- $f(0) = 0$
- $f'(x) > 0$ on $(-\infty, -3), (-3, -1)$ and $(0, \infty)$ $\quad$ $f'(x) < 0$ on (-1, 0)
- $f''(x) > 0$ on $(-\infty, -4), (-3, -1)$ and $(-1, \infty)$ $\quad$ $f''(x) < 0$ on (-4, -3)



2) (4 points) A ladder 15 feet long rests against a vertical wall, and the bottom edge slides away from the wall at a constant rate of $\frac{1}{4}$ ft/sec. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 12 feet from the wall?



$$x^2 + y^2 = 15^2 \qquad \frac{dx}{dt} = \frac{1}{4} \qquad \text{Want } \left.\frac{dy}{dt}\right|_{x=12}$$

$$y = \sqrt{15^2 - 12^2}$$

$$y = 9$$

$$\frac{d}{dt}\left(x^2 + y^2 = 225\right)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2 \cdot \overset{3}{\cancel{12}} \cdot \frac{1}{\cancel{4}} + 2 \cdot 9 \cdot \frac{dy}{dt} = 0$$

$$18\frac{dy}{dt} = -6$$

$$\boxed{\frac{dy}{dt} = \frac{-1}{3} \text{ ft/sec.}}$$

3) (3 points) Use calculus to find the absolute maximum and minimum of $f(x) = 3x^{2/3} - \frac{6}{5}x^{5/3}$ on the interval $[-1, 2]$.

$$f'(x) = 3 \cdot \frac{2}{3}x^{-1/3} - \frac{6}{5} \cdot \frac{5}{3}x^{2/3}$$

$$0 = \frac{2}{x^{1/3}} - 2x^{2/3} \cdot \frac{x^{1/3}}{x^{1/3}}$$

$$0 = \frac{2 - 2x}{x^{1/3}} = \frac{2(1-x)}{x^{1/3}}$$

$$f(-1) = 3 + \frac{6}{5} = 4.2$$

$$f(0) = 0$$

$\Rightarrow$ critical numbers $x = 1$ and $x = 0$

$$f(1) = 3 - \frac{6}{5} = 1.8$$

$$f(2) = 3\sqrt[3]{4} - \frac{6}{5}\sqrt[3]{32}$$

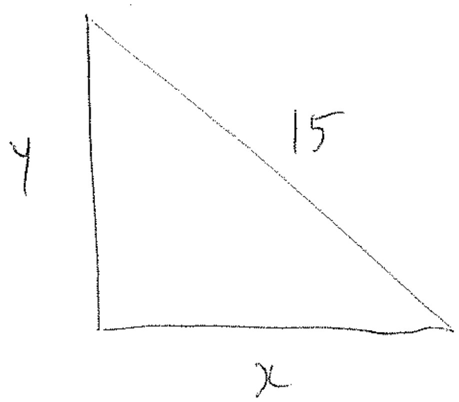
$$f(2) = 4.762 - 3.810 = 0.952$$

$\Rightarrow$ $\boxed{\text{Max of } 4.2, \text{ Min of } 0}$

Quiz 5B - MTH 1410

Name: Key

1) (4 points) A ladder 15 feet long rests against a vertical wall, and the top edge slides down the wall at a constant rate of $\frac{-1}{3}$ ft/sec. How fast is the ladder sliding away from the wall when the bottom of the ladder is 9 feet from the wall?



$$\frac{dy}{dt} = \frac{-1}{3}$$

Want $\left.\frac{dx}{dt}\right|_{x=9}$

$y = \sqrt{225-81} = 12$

$$\frac{d}{dt}\left(x^2+y^2=15^2\right)$$

$$\Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$2\cdot 9\cdot\frac{dx}{dt}+2(12)\left(\frac{-1}{3}\right)=0$$

$$18\frac{dx}{dt}=8$$

$$\frac{dx}{dt}=\frac{8}{18}=\boxed{\frac{4}{9}\text{ ft/sec}}$$

2) (3 points) Use calculus to find the absolute maximum and minimum of $f(x) = 3x^{2/3} - \frac{6}{5}x^{5/3}$ on the interval $[\frac{-1}{8}, 4]$.

$$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} - \frac{6}{5} \cdot \frac{5}{3} x^{2/3}$$

$$= \frac{2}{x^{1/3}} - \frac{2x^{2/3} \cdot x^{1/3}}{x^{1/3}}$$

$$f'(x) = \frac{2-2x}{x^{1/3}}$$

critical #s at $2-2x=0$

$x^{1/3} = 0$

$\Rightarrow x = 1, 0$

check: $f(\frac{-1}{8}) = \frac{3}{4} + \frac{6}{5 \cdot 32} = 0.7875$

$f(4) = 2.52 - 12.10 = -9.57$

$f(1) = 1.8$

$f(0) = 0$

$$\boxed{\begin{aligned} Max &= 1.8 \\ Min &= -9.57 \end{aligned}}$$

3) (3 points) Graph a *continuous* function f with the following properties:

- $f(0) = 0$ $\qquad f'(x) > 0$ on $(2,4)$ $\qquad f'(x) < 0$ on $(-\infty, 0), (0,2)$ and $(4, \infty)$
- $f''(x) > 0$ on $(-\infty, 0), (1,4)$ and $(4,\infty)$ $\qquad f''(x) < 0$ on (0,1)