Quiz #1A, MTH 1410

2:16
2:21
give 20 minutes

Name: Key

1. (4 points) For what value of c is the function continuous at $x = 4$? Make sure you explain your answer and use correct notation.

$$f(x) = \begin{cases} \frac{x^2-5x+4}{x-4} & \text{if } x \neq 4 \\ 3c & \text{if } x = 4 \end{cases}$$

Need $\lim_{x\to 4} f(x) = f(4)$ ✓ $\qquad f(4) = 3c$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4} \frac{x^2-5x+4}{(x-4)}$$

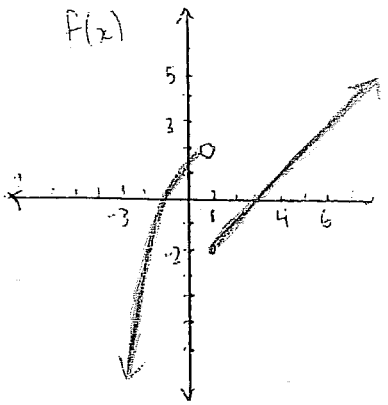
$$= \lim_{x\to 4} \frac{(x-4)(x-1)}{(x-4)}$$ ✓

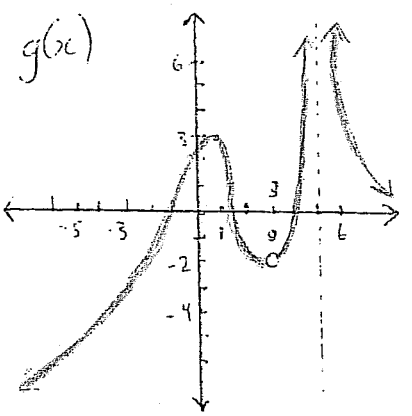
$$= 3$$

So we need $3 = 3c$ ✓

$$\Rightarrow \boxed{c = 1}$$ ✓

2. (2 points) For the given graphs, calculate the limit or state that it does not exist. If it does not exist, (briefly) explain why.





(i) $\lim_{x\to 1^+}[f(x)+g(x)] =$

$\lim_{x\to 1^+} f(x) = -2$ ✓

$\lim_{x\to 1^+} g(x) = 3$ ✓

$\Rightarrow \lim_{x\to 1^+} f(x)+g(x) = 3+(-2) = \boxed{1}$

(ii) $\lim_{x\to 3}[f(x)\cdot g(x)] =$

$\lim_{x\to 3} f(x) = 0$ ✓

$\lim_{x\to 3} g(x) = -2$ ✓

$\Rightarrow \lim_{x\to 3} f(x)\cdot g(x) = 0\cdot(-2) = 0$

3. (4 points) Calculate the limit, if it exists. If it does not exist, explain why. Hint: Combine fractions, and be sure to use correct notation.

$$\lim_{x\to 0}\frac{1}{x}\left(\frac{5}{7}-\frac{5}{x+7}\right)$$

$= \lim_{x\to 0}\frac{1}{x}\left(\frac{5(x+7)}{7(x+7)}-\frac{5\cdot 7}{(x+7)\cdot 7}\right)$ ✓

$= \lim_{x\to 0}\frac{1}{x}\left(\frac{5x+35-35}{7(x+7)}\right)$ ✓

$= \lim_{x\to 0}\frac{1}{x}\left(\frac{5x}{7(x+7)}\right)$ ✓

$= \lim_{x\to 0}\frac{5}{7(x+7)} = \boxed{\frac{5}{49}}$ ✓

Quiz #1B, MTH 1410

Name: Key

1. (4 points) Calculate the limit, if it exists. If it does not exist, explain why. Hint: Combine fractions, and be sure to use correct notation.

$$\lim_{x\to 0} \frac{1}{x}\left(\frac{3}{8} - \frac{3}{x+8}\right)$$

$$= \lim_{x\to 0} \frac{1}{x}\left(\frac{3(x+8)}{8(x+8)} - \frac{3(8)}{8(x+8)}\right) \checkmark$$

$$= \lim_{x\to 0} \frac{1}{x}\left(\frac{3x+24-24}{8(x+8)}\right) \checkmark$$

$$= \lim_{x\to 0} \left(\frac{1}{x}\right)\cdot\frac{3x}{8(x+8)} \checkmark$$

$$= \lim_{x\to 0} \frac{3}{8(x+8)} = \boxed{\frac{3}{64}} \checkmark$$

2. (4 points) For what value of c is the function continuous at $x = 1$? Make sure you explain your answer and use correct notation.

$$f(x) = \begin{cases} \frac{x^2-4x+3}{x-1} & \text{if } x \neq 1 \\ 2c & \text{if } x = 1 \end{cases}$$

$f(1) = 2c$

Need $\lim_{x \to 1} f(x) = f(1)$

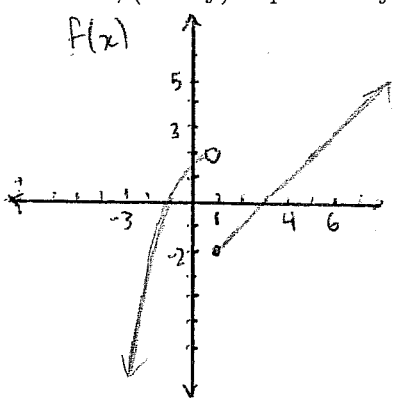
$$\lim_{x \to 1} \frac{x^2-4x+3}{x-1} = \lim_{x \to 1} \frac{(x-1)(x-3)}{(x-1)}$$

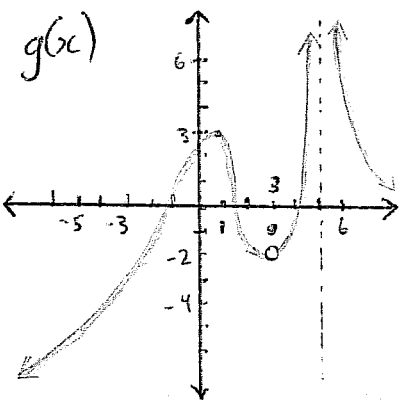
$$= \lim_{x \to 1} (x-3)$$

$$= -2$$

Need $2c = -2$ $\Rightarrow$ $c = -1$

3. (2 points) For the given graphs, calculate the limit or state that it does not exist. If it does not exist, (briefly) explain why.





(i) $\lim_{x \to 1^+} [f(x) + g(x)] = (-2) + 3 = 1$

$\lim_{x \to 1^+} f(x) = -2$, $\lim_{x \to 1^+} g(x) = 3$

(ii) $\lim_{x \to 5^-} [f(x) \cdot g(x)] =$

$\lim_{x \to 5^-} f(x) = 2$, $\lim_{x \to 5^-} g(x) = \infty$ (dne) so $\lim_{x \to 5^-} f(x) \cdot g(x)$ does not exist because one of the limits does not exist