Linear - Quiz 3

9:03
9:13
10 → give 20 minutes.

Name: Key

1. (4 points) Let A be a 3×4 matrix with 2 pivot positions. Answer the following and explain (briefly) your reasoning:

   (a) Does $A\mathbf{x} = 0$ have a nontrivial solution? $\boxed{\text{Yes}}$

$$\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If there are 2 pivot positions then there will be two free variables. Free variables $\Rightarrow$ nontrivial solutions. (ie. ∞ # of solutions)

   (b) Does $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible $\mathbf{b}$?

$\boxed{\text{No}}$. In reduced form, the augmented matrix $[A \,\vdots\, b]$ will be of the form $\left[\begin{array}{cccc|c} 1 & 0 & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & * \end{array}\right]$.

If b is such that this entry is $\neq 0$ then the system is inconsistent and there will be no solution.

2. (4 points) Find the value(s) of h for which the vectors are linearly dependent. Justify (briefly) your answer.

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix}$$

Ans: all values of h

So the 3rd column has no pivot and thus there is a free variable ⟹ the vectors will be linearly dependent no matter what the value of h.

3. (2 points) Let T be defined by $T(x) = A\mathbf{x}$, where $A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$. Find a vector $\mathbf{x}$ whose image under T is b=$\begin{bmatrix} 4 \\ 4 \end{bmatrix}$.

$$\left[\begin{array}{ccc|c} 1 & -5 & -7 & 4 \\ -3 & 7 & 5 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -5 & -7 & 4 \\ 0 & -8 & -16 & 16 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & -6 \\ 0 & 1 & 2 & -2 \end{array}\right]$$

$$\to x_1 + 3x_3 = -6$$

$$x_2 + 2x_3 = -2$$

Let $x_3 = 0$, get $x_1 = -6$, $x_2 = -2$

$$\Rightarrow \boxed{x = \begin{bmatrix} -6 \\ -2 \\ 0 \end{bmatrix} \text{ is an answer (there are an infinite \# of answers)}}$$

Quiz 2A - Math 1410

9:14
9:18
4 minutes
$\Rightarrow \boxed{\text{give 15 in class}}$

Name: Key

1) (6 points) Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{1}{2x+7}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \checkmark$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{2(x+h)+7} - \frac{1}{2x+7}\right) \quad \text{get common denom} \quad \checkmark$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\cancel{2x}+\cancel{7} - (\cancel{2x}+2h+\cancel{7})}{(2x+2h+7)(2x+7)}\right) \quad \checkmark$$

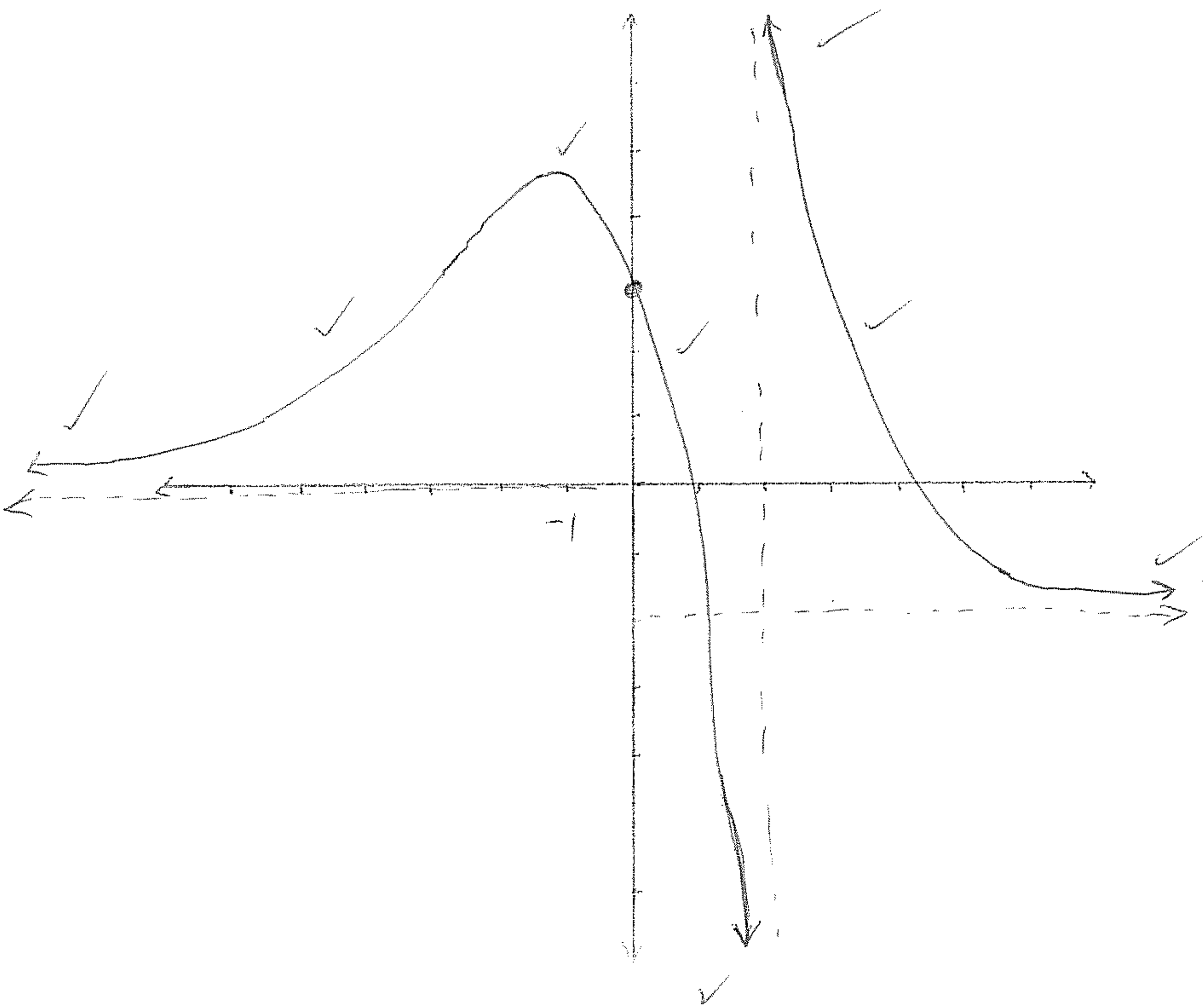
$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{-2\cancel{h}}{(2x+2h+7)(2x+7)}\right) \quad \checkmark$$

$$= \lim_{h \to 0} \frac{-2}{(2x+\cancel{2h}+7)(2x+7)} \quad \checkmark$$

$$= \frac{-2}{(2x+7)^2} \quad \checkmark$$

2) (4 points) Sketch a graph of the function $f(x)$ if the following conditions hold:

- $f(0) = 3$.
- $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = -2$.
- $\lim_{x\to 2^-} = -\infty$ and $\lim_{x\to 2^+} = \infty$.
- $f'(x) > 0$ on the interval $(-\infty, -1)$.
- $f'(x) < 0$ on the intervals $(-1, 2)$ and $(2, \infty)$.
- f has only one vertical asymptote.



Quiz 2B - Math 1410

Name: Key

1) (4 points) Sketch a graph of the function $f(x)$ if the following conditions hold:

• $f(0) = -3$.

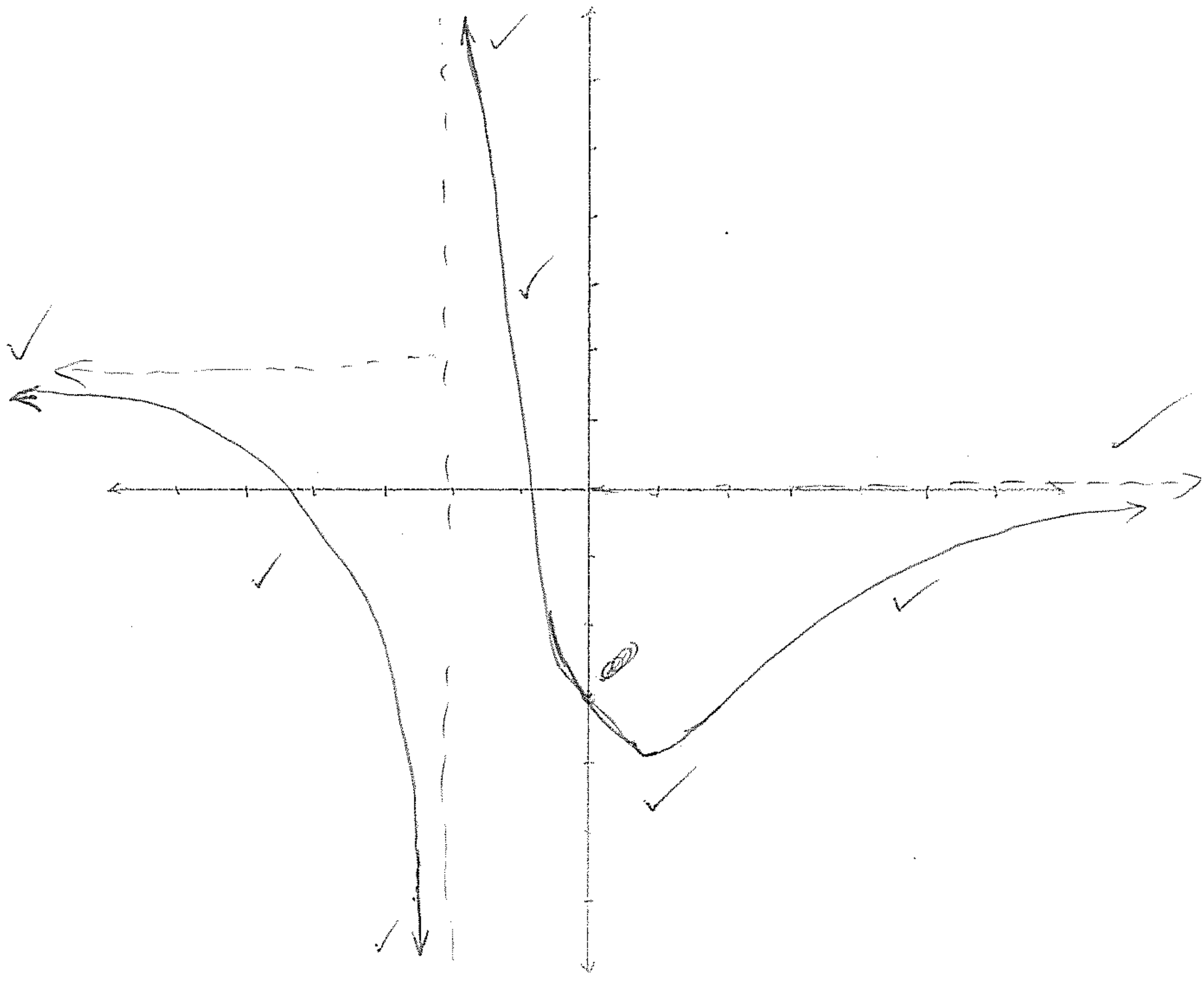
• $\lim_{x\to-\infty} f(x) = 2$ and $\lim_{x\to\infty} f(x) = 0$.

• $\lim_{x\to-2^-} = -\infty$ and $\lim_{x\to-2^+} = \infty$.

• $f'(x) < 0$ on the intervals $(-\infty, -2)$ and $(-2, 1)$.

• $f'(x) > 0$ on the interval $(1, \infty)$.

• f has only one vertical asymptote.



2) (6 points) Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{1}{3x+5}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{3(x+h)+5} - \frac{1}{3x+5} \right) \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x+5} - (\cancel{3x} + 3h + \cancel{5})}{(3x+3h+5)(3x+5)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{(3x+3h+5)(3x+5)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-3}{(3x+\cancel{3h}+5)(3x+5)}$$

$$= \frac{-3}{(3x+5)^2}$$