Test 2B, Math 152

2008

Name: Key

PID Number: ______________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will be counted wrong.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are allowed on this exam, but NOT cell phones or laptops.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge and write your PID on both pages.

Total # of question ... etc.

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Test B

1. (12 points) Find all **vertical and horizontal asymptotes**, if any, for the given functions:

(a) $f(x) = \dfrac{x^2+5x-6}{5x^2+20x-105}$

horiz: $\lim\limits_{x\to\infty} \dfrac{(x^2+5x-6)\cdot\frac{1}{x^2}}{(5x^2+20x-105)\cdot\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{1+\frac{5}{x}-\frac{6}{x^2}}{5+\frac{20}{x}-\frac{105}{x^2}}$

$= \frac{1}{5}$ (same for $\lim\limits_{x\to-\infty}$)

$\boxed{\text{horiz. asympt. is } y=\frac{1}{5}}$

vertical: $f(x) = \dfrac{(x+6)(x-1)}{5(x^2+4x-21)} = \dfrac{(x+6)(x-1)}{5(x+7)(x-3)}$

$\boxed{\Rightarrow \text{vertical asymptotes at } x=-7 \text{ and } x=3}$

(b) $f(x) = \dfrac{x^3}{x^2-2x}$

horiz: $\lim\limits_{x\to\infty} \dfrac{x^3\cdot\frac{1}{x^2}}{(x^2-2x)\cdot\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{x}{1-\frac{2}{x}} = \dfrac{\infty}{1-0} = \infty$

$\lim\limits_{x\to-\infty} = -\infty$

So no horiz. asymp.

vertical: $\dfrac{x^3}{x(x-2)}$ $\boxed{\text{So } x=2 \text{ is a vert. asymptote}}$

but $x=0$ is not since it is zero on top there as well.

2. (12 points) A raspberry field has an average yield of 160 berries per tree if the density of the bushes is 40 bushes/acre. For each unit increase in density (i.e. increase of one bush/acre), the yield decreases by 2 berries/bush. For example, at 41 bushes/acre the yield will only be 158 berries/bush. Use calculus to determine how many bushes should be planted to maximize the yield. Use the first or second derivative test (whichever you find appropriate) to verify that the yield is maximized.

$t =$ # of bushes greater than 40.

Total Yield = (# of bushes) · (Yield / bush)

$$Y(t) = (40+t)(160-2t)$$

$$Y'(t) = 1\cdot(160-2t) + (-2)(40+t)$$

$$= 160-2t-80-2t$$

$$Y'(t) = 80-4t \qquad Y''(t) = -4$$

$$0 = 80-4t$$

$$t = 20$$

We know $t = 20$ is a maximum since y'' is always negative $\Rightarrow$ concave down (2nd deriv. test).

Thus the # of bushes $= 40+20 = \boxed{60}$ to get a max yield.

3. (12 points) Given that $A = 1000\left(1 + \frac{r}{12}\right)^{60}$. This formula for A tells us the dollar value, after 5 years, of an account with an initial deposit of 1000 dollars and interest compounded monthly at the interest rate r. Suppose that the current rate is 6%; that is, $r = 0.06$. Use differentials to approximate the change in value of such an account if r were to increase from its current level to 0.062. Round your final answer to the nearest hundredth.

$$dA = A'(r) \cdot dr$$

$dr = 0.062 - 0.06$

$dr = 0.002$

$$A'(r) = 1000 \cdot 60 \cdot \left(1 + \frac{r}{12}\right)^{59} \cdot \frac{1}{12}$$

$$A'(r) = 5000\left(1 + \frac{r}{12}\right)^{59}$$

$$dA = 5000\left(1 + \frac{0.06}{12}\right)^{59} \cdot 0.002$$

$$= 10 \cdot (1.3421)$$

$$= 13.421$$

$$\boxed{\$13.42 \text{ increase}}$$

4. (12 points) A cannonball is shot into the air so that its height above the ground after t seconds is given by

$$h(t) = 26 + 210t - 7t^2$$

(a) At what value of t does the projectile reach maximum height? How do you know it is a maximum?

(6)

$h'(t) = 210 - 14t$

$h'(t) = 14(15 - t)$

$0 = 14(15-t) \Rightarrow t = 15$

1st deriv. test.

(3)

Check $t = 10$, $t = 20$

$h'(10) = 70$, $h'(20) = -70$

positive, negative

positive to negative $\Rightarrow$ max at $\boxed{t = 15}$

(b) What is the maximum height reached?

(3)

$h(15) = 26 + 210 \cdot 15 - 7(15)^2$

$= 26 + 3150 - 1575$

$= \boxed{1601}$

$$\begin{array}{r} 7 \\ 225 \\ \hline 1575 \end{array}$$

5. (12 points) The distance $d(t)$ covered by Speed Racer after t seconds (during a car race which starts at $t = 0$) is given by

$$d(t) = -t^4 - 6t^3 + 60t^2 + 5t$$

At what value of t *during the race* does the car change from accelerating to decelerating?

$d'(t) = -4t^3 - 18t^2 + 120t + 5$

$d''(t) = -12t^2 - 36t + 120$

$= -12(t^2 + 3t - 10)$

$d''(t) = -12(t+5)(t-2)$

not in domain

$d''(t) = 0$ at $t = -5$ and $t = 2$

Check $d''(1) = 72$ positive $\Rightarrow$ concave up $\Rightarrow$ accelerating

$d''(3) = -96$, negative $\Rightarrow$ concave down $\Rightarrow$ decelerating.

$\boxed{\text{at } t = 2 \text{ it changes}}$

6. (16 points) You are given that $m(x) = \dfrac{-x^2}{5(x^2+10)}$, $m'(x) = \dfrac{-4x}{(x^2+10)^2}$, $m''(x) = \dfrac{12(x^2-4)}{(x^2+10)^3}$. Use calculus to answer the following questions:

(a) Does the graph of m have a local maximum or minimum? If so, what are its coordinates? Max or Min when $m'(x)=0 \Rightarrow -4x=0 \Rightarrow x=0$

(4) at $x=0$, $m''(0)=\frac{-48}{1000}$ is negative $\Rightarrow$ $x=0$ is a max by 2nd derivative test. Note: $(x^2+10)^2>0 \Rightarrow m'(x)$ is defined everywhere.

Coordinates $(0, m(0)) = (0,0)$

(b) On what interval(s) is m decreasing?

m is decreasing when $m'(x)<0$. Test $m'(1)$, $m'(-1)$

(3) $m'(1) = \frac{-4}{121}$ negative, $m'(-1) = \frac{4}{121}$ positive. Thus m is decreasing on $(0,\infty)$

(c) On what interval(s) (if any) is the graph of m concave upward?

This will be when $m''(x)>0$. $m''(x)=0$ when

$12(x^2-4)=0 \Rightarrow x=+2$ or -2. Test $m''(-3)$, $m''(0)$, $m''(3)$

(4) $m''(-3) = \frac{12(5)}{19^3}$ is positive, $m''(0)=\frac{-48}{1000}$ is negative, $m''(3)=\frac{12(5)}{19^3}$ is pos.

So concave up on $(-\infty,-2)$ and $(2,\infty)$

(d) On what interval(s) (if any) is the graph of m concave downward?

(2) $(-2, 2)$ from above.

(e) Find the (x, y)-coordinates of each point of inflection for m. Answers *must* be written as ordered pairs. Inflection points at $x=2$ and -2.

(3) Coordinates $(2, \frac{-4}{70}) = \boxed{(2, \frac{-2}{35}) \text{ and } (-2, \frac{-2}{35})}$

7. (12 points) Sketch the graph of a continuous function f that has all of the following characteristics. Label all points that are Relative maximums, minimums, or inflection points:

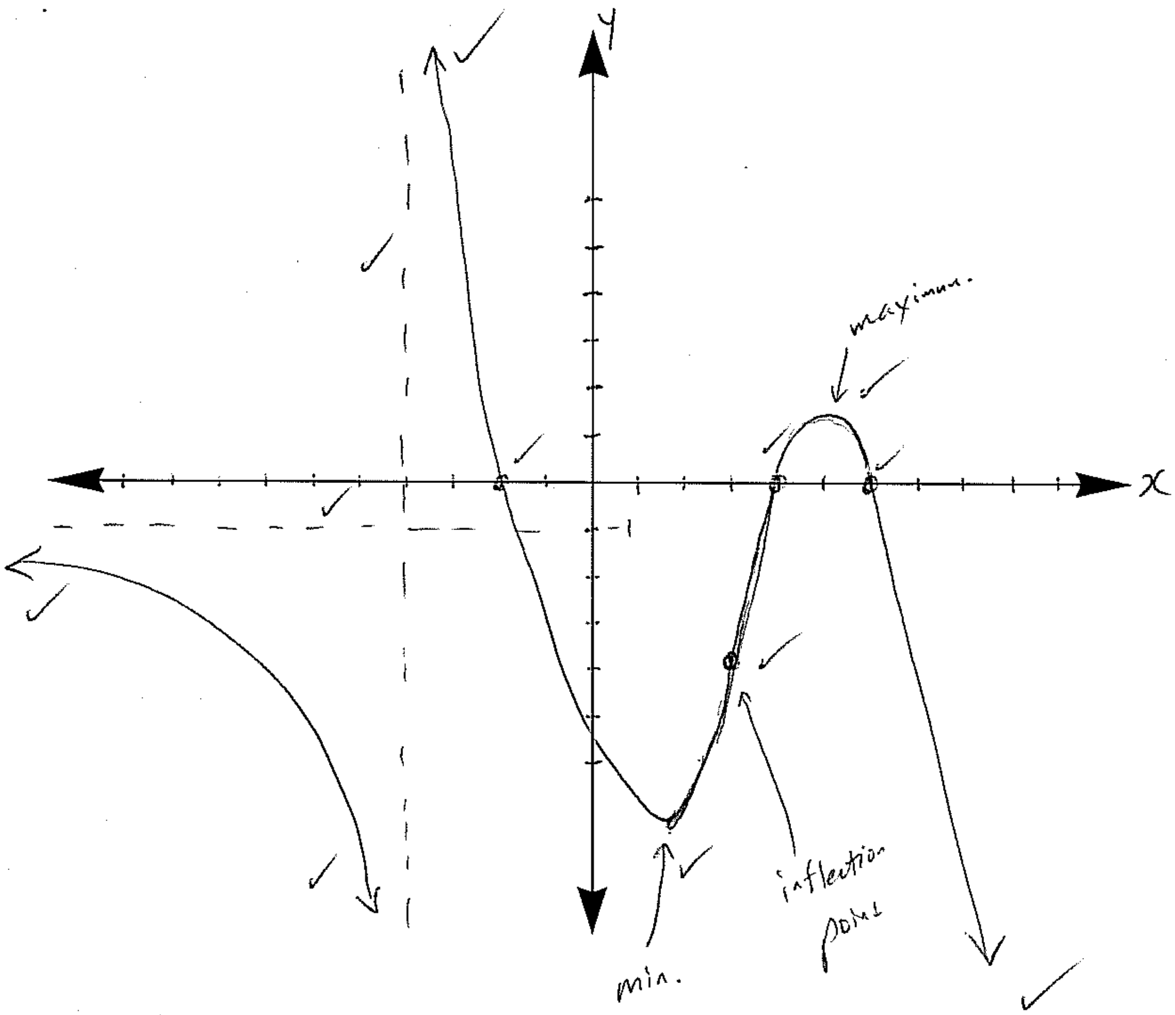
(a) $(-2,0)$, $(4,0)$ and $(6,0)$ are the only x-intercepts

(b) $\lim_{x\to(-\infty)} f(x) = -1$ and $\lim_{x\to\infty} f(x) = -\infty$

(c) The graph has a vertical asymptote at $x = -4$

(d) $f'(x) > 0$ on the interval $(2,5)$. On the intervals $(-\infty,-4)$, $(-4,2)$ and $(5,\infty)$ $f'(x) < 0$.

(e) On the interval $(-4,3)$, we have $f''(x) > 0$. On the intervals $(-\infty,-4)$ and $(3,\infty)$, $f''(x) < 0$.



8. (12 points) Use calculus to find the absolute maximum and absolute minimum values of $F(t) = t^3 - 147t$ on the interval $[-4, 10]$.

absolute maximum is 524 when $t =$ -4

absolute minimum is $-\cancel{8}86$ when $t =$ 7

$F'(t) = 3t^2 - 147 = 3(t^2 - 49) = 3(t-7)(t+7)$

$F'(t) = 0$ when $t = 7$ or -7

not in domain.

Compare $F(-4)$, $F(7)$, $F(10)$

$F(-4) = -64 + 588 = 524$

$F(7) = 343 - 1029 = -686$

$F(10) = 1000 - 1470 = -470$

$$\begin{array}{r} 147 \\ \times\ 7 \\ \hline 1029 \\ -343 \\ \hline 686 \end{array}$$

Extra Credit(2 points): Can an inflection point of a graph ever be a relative maximum or minimum? If yes give an example (either a function or a sketch of the graph). If no, explain why not.

No. Consider a minimum. On ~~one side~~ the left side f' is negative and on the right f' is positive. Thus f' is increasing on that region, so f'' cannot be zero (as it would for an inflection point).