Quiz 1A, Math 152, 2008

8:24 AM
8:33 AM

Name: Key

1) For the function $\dfrac{x^2-4x-21}{3x^2-6x-105}$, find:

(2) (a) $\displaystyle\lim_{x\to 7}\frac{x^2-4x-21}{3x^2-6x-105} = \lim_{x\to 7}\frac{(x-7)(x+3)}{3(x^2-2x-35)} = \lim_{x\to 7}\frac{\cancel{(x-7)}(x+3)}{3\cancel{(x-7)}(x+5)}$

$= \dfrac{10}{3(12)}$

$= \dfrac{10}{36}$

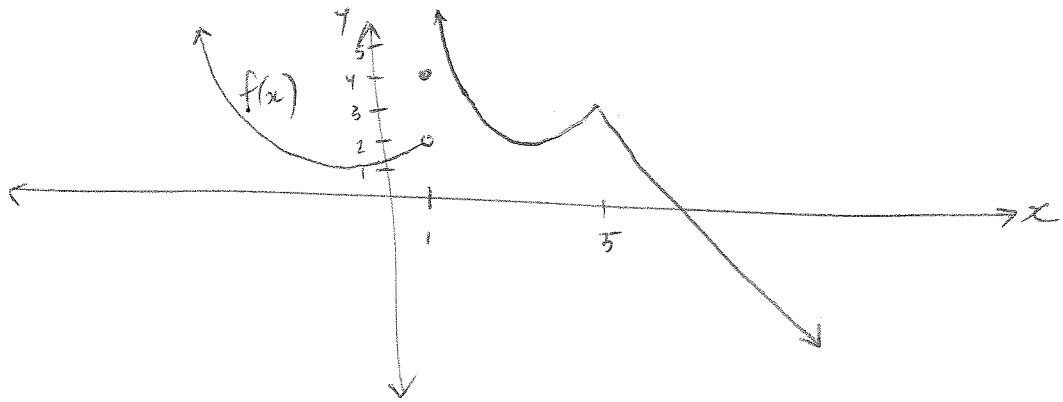
$\boxed{= 5/18}$

(1) (b) $\displaystyle\lim_{x\to\infty}\frac{x^2-4x-21}{3x^2-6x-105}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1-\frac{4}{x}-\frac{21}{x^2}}{3-\frac{6}{x}-\frac{105}{x^2}} = \frac{1-0-0}{3-0-0} = \boxed{\frac{1}{3}}$

(2) (c) $\displaystyle\lim_{x\to -5}\frac{x^2-4x-21}{3x^2-6x-105} = \lim_{x\to(-5)}\frac{(x-7)(x+3)}{3(x-7)(x+5)} = \frac{-12\cdot(-2)}{3\cdot(-12)(0)} = \frac{24}{0}$

is not defined. The limit $\boxed{\text{does not exist}}$ (One side goes to ∞, the other to $-\infty$).

2) Answer the questions for the following graph of $f(x)$:



(a) What is $\lim_{x \to 1^-} f(x)$?

(2) $= 2$ according to the graph

(b) Is f continuous at $x = 1$ and $x = 5$? If no, explain what part of continuity it fails.

(3) Not continuous at $x = 1$ since the limit does not exist ($\lim_{x \to 1^+} f(x) = \infty$, so the one-sided limits don't match up)

It is continuous at $x = 5$.

3) Use the 4-step process to find the derivative $f'(x)$ of $f(x) = 5x^2 + 2$.

① $f(x+h) = 5(x+h)^2 + 2 = 5(x^2 + 2xh + h^2) + 2$ (1)

$= 5x^2 + 10xh + 5h^2 + 2$

② $f(x+h) - f(x) = \cancel{5x^2} + 10xh + 5h^2 \cancel{+2} - (\cancel{5x^2} \cancel{+2})$ (1)

$= 10xh + 5h^2$

$= h(10x + 5h)$

③ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{h}(10x + 5h)}{\cancel{h}} = 10x + 5h$ (1)

④ $\lim_{h \to 0} 10x + 5h = \boxed{10x = f'(x)}$ (2)

Quiz 1B, Math 152, 2008

Name: Key

1) (5 points) Use the 4-step process to find the derivative $f'(x)$ of $f(x) = 3x^2 + 4$.

① $f(x+h) = 3(x+h)^2 + 4 = 3(x^2 + 2xh + h^2) + 4$
$= 3x^2 + 6xh + 3h^2 + 4$ (1)

② $f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 \cancel{+4} - (\cancel{3x^2 + 4})$
$= 6xh + 3h^2$ (1)
$= h(6x + 3h)$

③ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{h}(6x+3h)}{\cancel{h}} = 6x + 3h$ (1)

④ $\lim_{h \to 0} 6x + 3h = \boxed{6x = f'(x)}$ (2)

2) (5 points) For the function $\dfrac{x^2-2x-8}{5x^2+25x-180}$, find:

(a) $\displaystyle\lim_{x\to -9} \frac{x^2-2x-8}{5x^2+25x-180}$ $= \displaystyle\lim_{x\to -9} \frac{(x-4)(x+2)}{5(x^2+5x-36)}$

(2)

$= \displaystyle\lim_{x\to -9} \frac{(x-4)(x+2)}{5(x+9)(x-4)}$

$= \dfrac{(-13)(-7)}{5\cdot 0\cdot(-13)}$

$= \dfrac{91}{0}$ is not defined, $\Longrightarrow$ limit does not exist

In particular, $\displaystyle\lim_{x\to(-9)^+} = -\infty$, which is not a number

(b) $\displaystyle\lim_{x\to 4} \frac{x^2-2x-8}{5x^2+25x-180}$ $= \displaystyle\lim_{x\to 4} \frac{\cancel{(x-4)}(x+2)}{5(x+9)\cancel{(x-4)}}$

(2)

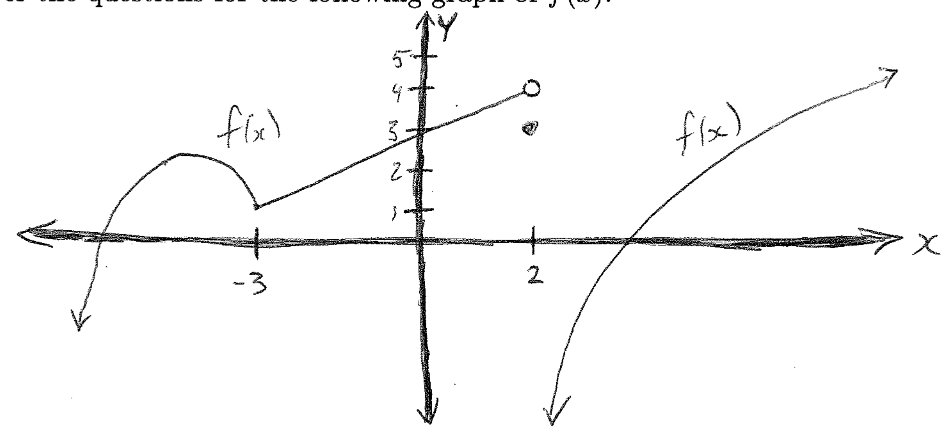
$= \dfrac{6}{5\cdot 13}$

$= \boxed{\dfrac{6}{65}}$

(c) $\displaystyle\lim_{x\to\infty} \frac{(x^2-2x-8)\cdot\frac{1}{x^2}}{(5x^2+25x-180)\cdot\frac{1}{x^2}}$ $= \displaystyle\lim_{x\to\infty} \frac{1-\frac{2}{x}-\frac{8}{x^2}}{5+\frac{25}{x}-\frac{180}{x^2}} = \frac{1-0-0}{5+0-0} = \boxed{\frac{1}{5}}$

(1)

3) (5 points) Answer the questions for the following graph of $f(x)$:



(a) What is $\lim_{x \to 2^-} f(x)$?

(2)

4, since that is the y-value being approached from the left as $x \to 2$.

(b) Is f continuous at $x = 2$ and $x = -3$? If no, explain what part of continuity it fails.

(3)

- f is continuous at $x = -3$.
- f is not continuous at $x = 2$ since the limit does not exist. Namely, $\lim_{x \to 2^+} = -\infty$ so the limit cannot exist.