

Business Calculus Test 1 Review

Dr. Graham-Squire, Summer Session 1, 2012

1. Factor and simplify the expression $5x^2(3x^2 + 1)^4(6x) + (3x^2 + 1)^5(2x)$.

Ans: $2x(3x^2 + 1)^4(18x^2 + 1)$

2. The average speed of a vehicle in miles per hour over a certain stretch of road, between 6 AM and 8 AM, is given by

$$20t^2 - 40t + 50 \quad (0 \leq t \leq 2)$$

where t is in hours, with $t = 0$ corresponding to 6 AM. Over what interval(s) of time is the average speed less than or equal to 35 mph?

Ans: $\frac{1}{2} \leq t \leq \frac{3}{2}$

3. Dominic has decided to attach his baby sister Eva to a rocket. Suppose Dominic is standing 200ft from the rocket (over flat ground) and the rocket with Eva attached lifts off vertically in the air and reaches an altitude of x feet.

(a) Find an expression (in terms of x) giving the distance between Dominic and Eva as the rocket goes up.

(b) What is the distance between Dominic and Eva when the rocket reaches an altitude of 20,000 feet?

Ans: (a) $d = \sqrt{x^2 + 40,000}$. (b) approximately 20,001 feet.

4. Find the equation of the line that passes through the points (1,2) and (-3,-2), and write the equation in the form $y = \underline{\hspace{2cm}}$. Is the point (4, 8) also on that line? Explain why or why not.

Ans: $y = x + 1$. No, the point (4, 8) is not on the line because $4+1=5 \neq 8$.

5. Sketch the graph of the function $f(x) = \begin{cases} 4 - x & \text{if } x < 2 \\ 2x - 2 & \text{if } x \geq 2 \end{cases}$

Ans: Graph looks like a V with the point at $x = 2$.

6. Find the rules for the composite functions $f(g(x))$ and $g(f(x))$ if $f(x) = \frac{x}{x^2 + 5}$ and $g(x) = \frac{1}{x^2}$. Simplify the resulting expressions.

Ans: $f(g(x)) = \frac{x^2}{5x^4 + 1}$ and $g(f(x)) = \frac{x^4 + 10x^2 + 25}{x^2}$.

7. The sales of DVD players in year t (in millions of units) is given by the function

$$f(t) = 5.6(1 + t) \quad (0 \leq t \leq 3)$$

where $t = 0$ corresponds to 2001. Over the same period, the sales of VCRs (in millions of units) is given by

$$g(t) = \begin{cases} -9.6t + 22.5 & \text{if } 0 \leq t \leq 1 \\ -0.5t + 13.4 & \text{if } 1 < t \leq 2 \\ -7.8t + 28 & \text{if } 2 < t \leq 3 \end{cases}$$

- (a) Show that more VCRs than DVD players were sold in 2001.
(b) When did sales of DVD players first exceed those of VCRs?

Ans: (a) $f(0) = 5.6$ and $g(0) = 22.5$, so there were more VCRs at $t = 0$, which is 2001. (b) Setting $f(t)$ equal to the middle equation of $g(t)$ solves to $t = 1.28$. So DVDs first exceeded VCRs in early 2002.

8. Farmer Bob wants to enclose a rectangular area next to his barn with fencing. One side of the rectangular area will be the side of the barn, and the other three sides will be fencing. If Bob has 80 feet of fencing, his barn is 100 feet long, and the width of the rectangular area is x , find an expression for the area of the rectangle. If the width is 10 feet, what will the area be?

Ans: $A(x) = x(80 - 2x)$ and $A(10) = 600ft^2$.

9. Find the limits. If the limit does not exist, write DNE and explain why.

(a) $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = 4$

(b) $\lim_{x \rightarrow (-1)} \frac{x^2}{x + 1}$. DNE because you are getting a number divided by zero.

(c) $\lim_{x \rightarrow \infty} \frac{3x^4 - 3x}{7x^2 - 11x^4 + 4} = -3/11$

10. Let $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

Find the value of the following limits. If the limit does not exist, write DNE and explain why.

(a) $\lim_{x \rightarrow (-1)^-} f(x) = 1$

(b) $\lim_{x \rightarrow (-1)^+} f(x) = 1$

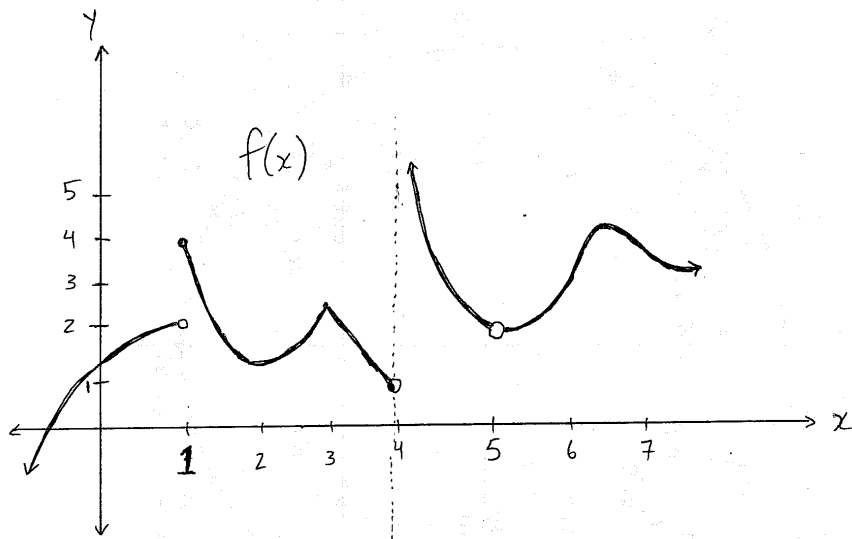
(c) $\lim_{x \rightarrow 2^-} f(x) = 4$

(d) $\lim_{x \rightarrow 2^+} f(x) = 3$

(e) $\lim_{x \rightarrow 1^-} f(x) = 1$

(f) $\lim_{x \rightarrow \infty} f(x) = 3$

11. Find the given limits for the following diagram. If the limit does not exist, write DNE and explain why.



- (a) $\lim_{x \rightarrow 1^+} f(x) = 4$
(b) $\lim_{x \rightarrow 4^-} f(x) = 1$
(c) $\lim_{x \rightarrow 4^+} f(x) = \text{DNE}$ because the graph is going to infinity.
(d) $\lim_{x \rightarrow 5} f(x) = 2$