

Quiz 5A, Business Calculus

Spring 2017 - Dr. Graham-Squire

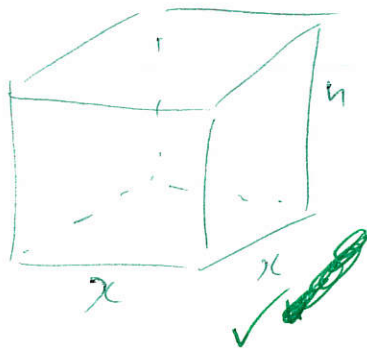
6:15

⇒ 25 minutes.

Name: Key

- (5 points) Aya wants to build a coop for her backyard chickens. It will be a large box, with a square bottom. She does not need any material for the bottom (since it will be on the ground), but the chain link side fencing will cost her \$5 per square foot and the roof will cost \$30 per square foot. She has exactly \$2000 to build the chicken coop. What is the maximum volume she can create for the coop?
 - Draw a diagram of the situation, and label any pertinent variables.
 - Write an equation to represent the cost of the coop and an equation to represent the volume.
 - Use calculus to maximize the volume of the coop.

(a)



$$\text{Volume} = x^2 h \quad \checkmark$$

~~Cost = 5(4xh) + 30(x^2)~~

$$2000 = 5(4xh) + 30(x^2) \quad \checkmark \checkmark$$

$$\Rightarrow \frac{2000 - 30x^2}{20x} = \frac{20xh}{20x}$$

$$\frac{100}{x} - \frac{3x}{2} = h$$

$$V(x) = x^2 \left(\frac{100}{x} - \frac{3x}{2} \right) \quad \checkmark$$

$$V(x) = 100x - \frac{3x^3}{2} \quad \checkmark$$

$$V'(x) = 100 - \frac{9}{2}x^2 \quad \checkmark$$

$$\checkmark 0 = 100 - \frac{9}{2}x^2$$

$$100 = \frac{9}{2}x^2$$

$$\sqrt{\frac{200}{9}} = \sqrt{x^2}$$

$$x = \frac{10\sqrt{2}}{3} \approx 4.714 \quad \checkmark$$

$$\Rightarrow V(4.714) = 314.27$$

↓
okay if check max instead.

2. (2 points) Use the Laws of Logarithms to expand and simplify the expression:

$$\begin{aligned} & \ln \frac{x^5}{\sqrt{x}(1+x)^7} \\ &= \ln x^5 - \ln (\sqrt{x} (1+x)^7) \\ &= \ln x^5 - (\ln \sqrt{x} + \ln (1+x)^7) \\ &= 5 \ln x - \ln x^{1/2} - 7 \ln (1+x) \\ &= 5 \ln x - \frac{1}{2} \ln x - 7 \ln (1+x) \\ &\text{or } \boxed{4.5 \ln x - 7 \ln (1+x)} \end{aligned}$$

3. (3 points) Bob wants to invest \$1000 in a savings account that is compounded continuously. If he gets a 5% interest rate, how many years will it take for his investment to grow to \$1700? Round your answer to the nearest 0.1 years.

$$\begin{aligned} & \checkmark A = Pe^{rt} \qquad A = 1700, P = 1000, r = 0.05 \\ & \frac{1700}{1000} = \frac{1000 e^{0.05t}}{1000} \checkmark \checkmark \\ & \ln(1.7) = \ln(e^{0.05t}) \checkmark \\ & \ln(1.7) = 0.05t \\ & \frac{\ln(1.7)}{0.05} = t \qquad \boxed{t = 10.6 \text{ years}} \end{aligned}$$