Test 3A - MTH 1310

Dr. Graham-Squire, Fall 2014

8:06
8:21
15

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

______________________________ (*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first — questions of the test, however you should still show all of your work. No calculators are allowed on the last — questions.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
6. Make sure you sign the pledge.
7. Number of questions = 7. Total Points = 50.

1. (6 points) In 1944, the Dow Jones Industrial Average stock market had an average of about 200. In 2014, it had grown to approximately 17,000. Assuming a continuously compounding model, what was the average percentage rate of return on an investment between 1944 and 2014? Round your answer to the nearest tenth of a percent.

$A = Pe^{rt}$ $\quad P = 200$ $\quad t = 70$ is 2014

$t = 0$ is 1944

$17{,}000 = 200e^{r(70)}$

$\frac{17{,}000}{200} = e^{70r}$

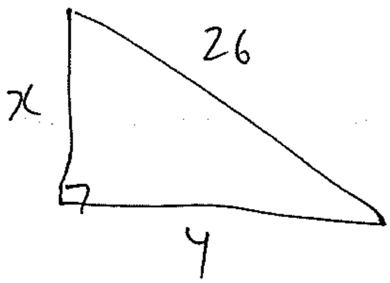
$\ln(85) = \cancel{\ln}e^{70r}$

$\frac{\ln 85}{70} = r$

$r = 0.0634 \Rightarrow \boxed{6.3\%}$

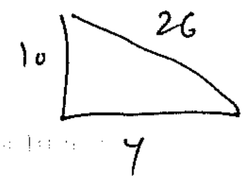
2. (6 points) A 26 foot ladder is leaning against a wall, but its feet start to slip and it begins to slide down the wall. Suppose that when the top of the ladder is 10 feet from the ground, the top of the ladder is sliding down the wall at a speed of 3 feet/sec. How fast is the *bottom* of the ladder sliding away from the wall?

At that moment



$$\frac{dx}{dt} = -3 \quad \text{when} \quad x = 10$$

$$\text{Want } \frac{dy}{dt}$$



$$\frac{d}{dt}\left(x^2 + y^2 = 26^2\right)$$

$$10^2 + y^2 = 26^2$$

$$y^2 = 676 - 100$$

$$y = \sqrt{576}$$

$$y = 24$$

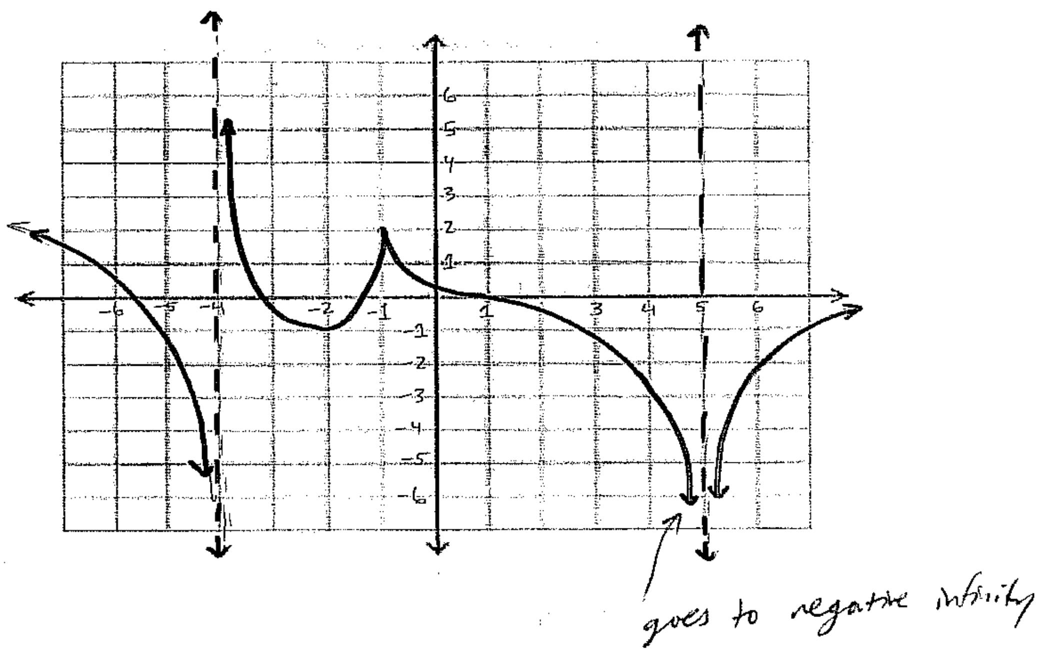
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(10)(-3) + 2(24)\frac{dy}{dt} = 0$$

$$48\frac{dy}{dt} = 60$$

$$\boxed{\frac{dy}{dt} = \frac{60}{48} = \frac{5}{4} = 1.25 \text{ ft/sec}}$$

3. (10 points) Below is the graph of $f(x)$. Answer the questions at the bottom of the page.



(a) On what interval(s) is f increasing? $(-2,-1)$ and $(5,\infty)$

(b) On what intervals(s) is f concave down?

$(-\infty,-4)$, $(1,5)$, $(5,\infty)$

(c) Find the absolute maximum and absolute minimum on the interval [-3,5). If it does not exist, write DNE and briefly explain why.

Abs. max of 2 (at $x=-1$), abs min DNE b/c $\lim_{x\to 5^-} f(x) = -\infty$

(d) Find the (x, y)-coordinate(s) of all inflection point(s).

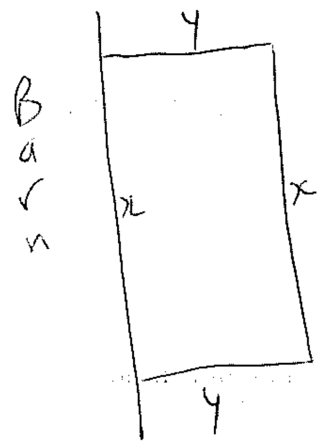
$(1,0)$

(e) Find an x-value at which the graph changes from decreasing to increasing, but is NOT a local minimum.

$x=5$

4. (8 points) Hank wants to build a rectangular garden on the side of his barn. For the part of the garden along his barn, he needs to put up wood slats to keep the dirt off of his awesome paint job. The wood slats cost \$2 per foot. On the other three sides he is putting up an electric fence to zap any bunnies that try to eat his lettuce. The electric fencing costs \$5 per foot. Assuming Hank has only \$800 total to spend on the fencing and wood slats, what is the largest area he can make for his garden?

Round to nearest whole #



$$2x + 5x + 2(5y) = \text{Cost}$$

$$7x + 10y = 800$$

$$\frac{10y}{10} = \frac{800 - 7x}{10}$$

$$y = 80 - \frac{7}{10}x$$

$$\text{Area} = xy$$

$$A(x) = x\left(80 - \frac{7}{10}x\right)$$

$$A(x) = 80x - \frac{7}{10}x^2$$

$$A'(x) = 80 - 1.4x \rightarrow A''(x) = -1.4 \text{ is conc. down} \Rightarrow \text{maximum}$$

$$0 = 80 - 1.4x$$

$$x = \frac{80}{1.4} = 57.14$$

$$A(57.14) = \boxed{2286 \text{ ft}^2}$$

Test A NO CALCULATOR

Name: Key

5. (6 points) Use logarithmic differentiation to calculate the derivative of

$$f(x) = (x^2 + 4)^{\ln x}$$

$$y = (x^2+4)^{\ln x}$$

$$\ln y = \ln (x^2+4)^{\ln x}$$

$$\frac{d}{dx}\left(\ln y = (\ln x)(\ln (x^2+4))\right)$$

$$\frac{y'}{y} = \frac{1}{x}\ln(x^2+4) + \ln x \cdot \frac{1}{x^2+4}\cdot 2x$$

$$y' = y\left(\frac{\ln(x^2+4)}{x} + \frac{2x\ln x}{x^2+4}\right)$$

$$\boxed{y' = (x^2+4)^{\ln x}\left(\frac{\ln(x^2+4)}{x} + \frac{2x\ln x}{x^2+4}\right)}$$

6. (8 points) Let $f(x) = \frac{4}{x^2} + x$. Use calculus to find

$f(x) = 4x^{-2} + x$

(a) The interval(s) on which f is increasing.

(b) The interval(s) on which f is decreasing.

(c) The (x, y)-coordinates of all local maximums and minimums, if any exist.

$f'(x) = -8x^{-3} + 1$

$f'(x) = \frac{-8}{x^3} + 1$ dne @ $x = 0$

$0 = \frac{-8}{x^3} + 1$

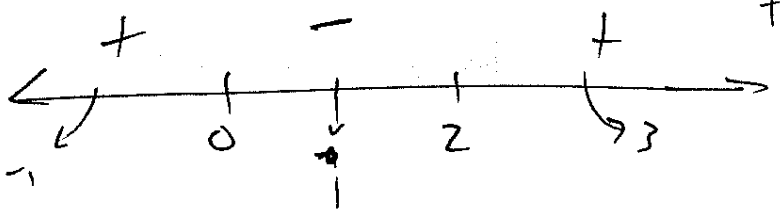
$\frac{8}{x^3} = 1$

$\sqrt[3]{x^3} = \sqrt[3]{8} \Rightarrow x = 2$

$f'(-1) = 9 \Rightarrow +$

$f'(1) = -7 \Rightarrow -$

$f'(3) = \frac{-8}{27} + 1 \Rightarrow +$



(a) $(-\infty, 0), (2, \infty)$

(b) $(0, 2)$

(c) @ $x = 0$, $f(x)$ dne so no maximum

@ $x = 2$, $y = \frac{4}{4} + 2 = 3 \Rightarrow$ $(2,3)$ is a local minimum

7. (6 points) Calculate the derivative of

$$f(x) = \left(e^{2x^2+7} + x^3\right)^6$$

You do not need to simplify your answer.

$$\boxed{f'(x) = 6\left(e^{2x^2+7} + x^3\right)^5\left(e^{2x^2+7}(4x) + 3x^2\right)}$$

Extra Credit (2 points) Use logarithmic differentiation to prove $\frac{d}{dx}(4^x) = (4^x)(\ln 4)$.

$$y = 4^x$$

$$\ln y = \ln 4^x$$

$$\frac{d}{dx}\left(\ln y = x(\ln 4)\right)$$

$$\frac{y'}{y} = \ln 4$$

$$y' = y \cdot (\ln 4)$$

$$\boxed{y' = 4^x(\ln 4)}$$