Business Calculus - Test 2 Review

Dr. Graham-Squire, Fall 2014

1. Find the limits. If the limit does not exist, write DNE and explain why.

   (a) $\lim_{x\to 2} \frac{x^2+4x-12}{x^2-2x}$=4

   (b) $\lim_{x\to(-1)} \frac{x^2}{x+1}$. DNE because you are getting a number divided by zero.

   (c) $\lim_{x\to\infty} \frac{3x^4-3x}{7x^2-11x^4+4}$= -3/11

2. Let $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

   Find the value of the following limits. If the limit does not exist, write DNE and explain why.

   (a) $\lim_{x\to(-1)^-} f(x)$=1

   (b) $\lim_{x\to(-1)^+} f(x)$=1

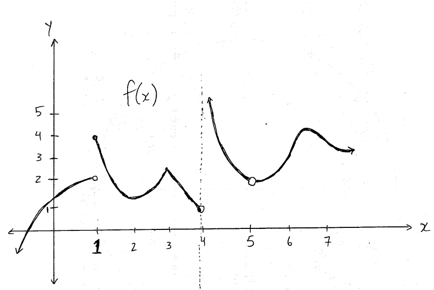
   (c) $\lim_{x\to 2^-} f(x)$=4

   (d) $\lim_{x\to 2^+} f(x)$=3

   (e) $\lim_{x\to 1^-} f(x)$=1

   (f) $\lim_{x\to\infty} f(x)$=3

3. Find the given limits for the following diagram. If the limit does not exist, write DNE and explain why.



   (a) $\lim_{x\to 1^+} f(x) = 4$

   (b) $\lim_{x\to 4^-} f(x) = 1$

   (c) $\lim_{x\to 4^+} f(x) =$ DNE because the graph is going to infinity.

   (d) $\lim_{x\to 5} f(x) = 2$

4. Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = \dfrac{1}{2x+3}$.

   Ans: You need to do $f(x+h) =$, etc. You answer should be $f'(x) = \dfrac{-2}{(2x+3)^2}$.

5. Find the derivative of each function: NOTE: Some of these answers are in simplified form

   (a) $f(x) = (3x^4 - 7)(x^2 + 9)$. Use the product rule to get $18x^5 + 108x - 14x$

   (b) $f(x) = (x^3 - 7x + 9)^7$. Chain rule: Ans: $7(x^3 - 7x + 9)^6(3x^2 - 7)$

   (c) $f(x) = \left(\dfrac{x^3 - 9}{x + 4}\right)^3$. Ans: $\dfrac{3(x^3 - 9)^2(2x^3 + 12x^2 + 9)}{(x+4)^4}$

   (d) $f(x) = (x + 7)^4(3x^2 - 4)^2$. Ans: $4(x + 7)^3(3x^2 - 4)(6x^2 + 21x - 4)$

6. The quantity x of TV sets demanded each week is related to the wholesale price by the equation $p = -0.006x + 180$. The weekly total cost for producing x sets is given by $C(x) = 0.00002x^3 - 0.02x^2 + 120x + 60,000$.

   (a) Find the revenue function $R(x)$ and the profit function $P(x)$.

   Ans: $R(x) = -0.006x^2 + 180x$, $P(x) = -0.00002x^3 + 0.014x^2 + 60x - 60,000$.

   (b) Compute the marginal revenue, cost, and profit functions.

   Ans: $R'(x) = -0.012x + 180$, $C'(x) = 0.00006x^2 - 0.04x + 120$, $P'(x) = -0.00006x^2 + 0.028x + 60$

   (c) Compute $R'(2000)$, $C'(2000)$, and $P'(2000)$ and interpret your results. What does that information tell the company about how many TV sets they should produce?

   Ans: $R'(2000) = 156$, $C'(2000) = 280$, and $P'(2000) = -124$. At production of 2000 TV sets, the costs still exceed the revenues and the next TV made will not give any profit. Producing only 2000 TV sets is not good for profits.

7. The number of people receiving disability benefits from 1990 through 2000 is approximated by the function

   $$N(t) = 0.00037t^3 - 0.0242t^2 + 0.52t + 5.3 \qquad (0 \le t \le 10)$$

   where $N(t)$ is measured in units of a million and t is measured in years with $t = 0$ being 1990. Compute $N(8)$, $N'(8)$, and $N''(8)$ and interpret your results. What does that information tell you about what was happening with disability benefits at that time, and what might it imply for the future?

   Ans: 8.1 million, 203,840, and -30,640. This means that in 1998, 8.1 million people were receiving disability benefits, and that number was increasing by 203,840 a year. Since the second derivative is negative, though, the amount of the increase is going to go down in the next year, by about 30,640 people. So even though the number of people receiving disability benefits will be increasing, the amount of the increase was going down in 1998.

8. Find $\dfrac{dy}{dx}$ for the equation $x^3 + xy^2 + y^3 = 0$.

   Ans: $\dfrac{dy}{dx} = \dfrac{-3x^2 - y^2}{2xy + 3y^2}$