Test 2A - MTH 1310

Dr. Graham-Squire, Fall 2014

Name: Key

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8:27
16

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are not allowed on this test. Calculators are allowed on the first — questions of the test, however you should still show all of your work. No calculators are allowed on the last — questions.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
6. Make sure you sign the pledge.
7. Number of questions = 9. Total Points = 50.

1. (6 points) A niche car company estimates their profit function to be

$$P(x) = 10,057x - \frac{8}{3}x^3 - 20,000.$$

Where x is the number of cars made and P is in dollars.

(a) Calculate the Marginal Profit function.

(b) Calculate $P'(20)$ and $P'(40)$.

(c) What would be an optimum level of production? Explain your answer, and make sure you reference the marginal profit function in order to get full points.

(a) $P'(x) = 10,057 - \frac{8}{3}\cdot 3x^2 - 0$

$$\boxed{P'(x) = 10,057 - 8x^2}$$

(b) $P'(20) = 6857$

$P'(40) = -2743$

(c) Based on (b.), should be between 20 and 40.

Want to have $P'(x) = 0$ to be sure we have gotten as much profit as possible.

Set $0 = 10,057 - 8x^2$

$$\sqrt{\frac{10,057}{8}} = \sqrt{x^2}$$

$$35.45 = x$$

$\Rightarrow$ Make only 36, because $P'(36)$ is negative $\Rightarrow$ 37^{th} car is losing profit

2. (5 points) Use the 4-step process (the limit definition of the derivative) to calculate the derivative of $f(x) = 5x^2 + 1$. You can check your work by using the Power Rule, but you have to use the 4-step process to get the answer in order to get full credit.

(1) $f(x+h) = 5(x+h)^2 + 1$

$= 5(x^2 + 2xh + h^2) + 1$

$= 5x^2 + 10xh + 5h^2 + 1$

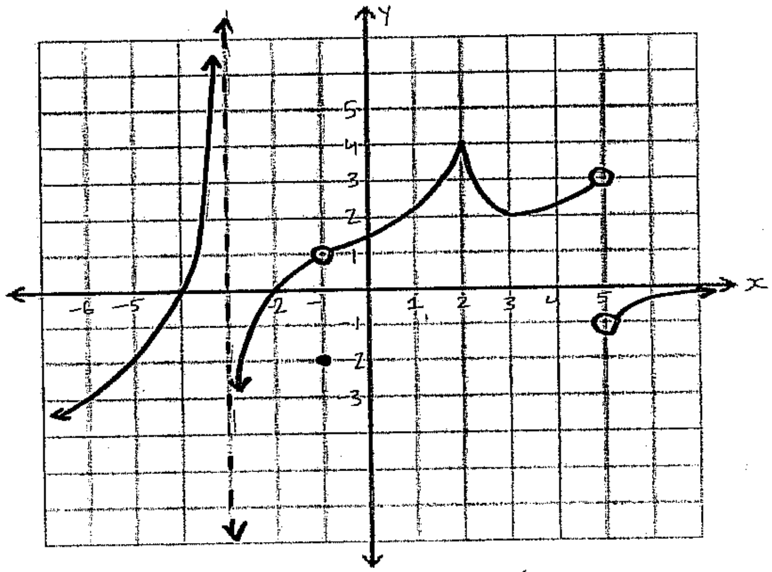
(2) $f(x+h) - f(x) = \cancel{5x^2} + 10xh + 5h^2 \cancel{+1} - (\cancel{5x^2} \cancel{+1})$

$= 10xh + 5h^2$

(3) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{10xh + 5h^2}{h} = \dfrac{\cancel{h}(10x + 5h)}{\cancel{h}} = 10x + 5h$

$f'(x) = \lim_{h \to 0} (10x + 5h) = 10x + 0 = \boxed{10x}$

3. (6 points) Consider the graph of $f(x)$ below, then answer the questions.



Calculate:

(a) $\lim_{x \to (-1)} f(x)$ = 1

(b) $\lim_{x \to 5^-} f(x)$ = 3

(c) $f(2)$ = 4

Find an x-value where each of the following occur. You have to use a different x-value for each one:

(d) An x-value where $f(x)$ is discontinuous:

$x = -3$, $x = -1$, or $x = 5$

(e) An x-value where $f(x)$ exists but $f'(x)$ does not exist:

$x = -1$ or $x = 2$

(f) An x-value where $f'(x) = 0$:

$x = 3$

4. (4 points) Calculate the limit. Make sure to show your work and use appropriate notation to get full points!

$$\lim_{x\to(-\infty)} \frac{\left(2x^3 - 7x + 4\right)}{\left(-3x^4 + 11x^2\right)} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}}$$

$$= \lim_{x\to(-\infty)} \frac{\frac{2x^3}{x^4} - \frac{7x}{x^4} + \frac{4}{x^4}}{\frac{-3x^4}{x^4} + \frac{11x^2}{x^4}}$$

$$= \lim_{x\to(-\infty)} \frac{\frac{2}{x} - \frac{7}{x^3} + \frac{4}{x^4}}{-3 + \frac{11}{x^2}}$$

$$= \frac{0 - 0 + 0}{-3 + 0} = \frac{0}{-3} = \boxed{0}$$

Name: Key

5. (6 points) Calculate $f''(x)$ if $f(x) = \sqrt{x^5 + 6}$. You do not need to simplify your answer.

$$f(x) = (x^5+6)^{1/2}$$

$$f'(x) = \frac{1}{2}(x^5+6)^{-1/2} \cdot 5x^4$$

$$f'(x) = \left(\frac{5}{2}x^4\right)(x^5+6)^{-1/2}$$

$$f''(x) = 4 \cdot \frac{5}{2}x^3 \cdot (x^5+6)^{-1/2} + \left(\frac{5}{2}x^4\right) \cdot \left(-\frac{1}{2}(x^5+6)^{-3/2} \cdot 5x^4\right)$$

$$= \boxed{10x^3 \cdot (x^5+6)^{-1/2} - \frac{25}{4}x^8 (x^5+6)^{-3/2}}$$

6. (5 points) Calculate the derivative below. You do not need to simplify your answer.

$$\frac{d}{dx}\left(\frac{(x^2+3)(x-7)}{3x^2-4}\right)^8$$

$$= 8\left(\frac{(x^2+3)(x-7)}{3x^2-4}\right)^7 \cdot \left(\frac{(3x^2-4)\left[(x^2+3)(1)+2x(x-7)\right]-6x(x^2+3)(x-7)}{(3x^2-4)^2}\right)$$

7. (6 points) Find the tangent line to the given function at $x = 1$. Make sure to show your work!

$$f(x) = \frac{x^6 + 4x^3 + x}{x^3}$$

$$f(x) = \frac{x^6}{x^3} + \frac{4x^3}{x^3} + \frac{x}{x^3}$$

$$f(x) = x^3 + 4 + x^{-2}$$

$$f'(x) = 3x^2 + 0 - 2x^{-3}$$

$$f'(1) = 3 - 2 = \underset{m}{1} \qquad f(1) = \frac{1+4+1}{1} = \overset{y_1}{6}$$

Tangent Line: $y - y_1 = m(x - x_1)$

$$\boxed{y - 6 = 1(x - 1)}$$

$$\boxed{y = x + 5}$$

8. (6 points) Let $f(x) = \begin{cases} \dfrac{x^2-3x-10}{x+2} & \text{if } x \le 3 \\ 2x+1 & \text{if } x > 3 \end{cases}$

Calculate the following limits:

(a) $\lim_{x \to (-2)} f(x) = \lim_{x \to (-2)} \dfrac{x^2-3x-10}{x+2}$

$= \lim_{x \to (-2)} \dfrac{(x-5)\cancel{(x+2)}}{\cancel{x+2}}$

$= \lim_{x \to (-2)} x-5 = -2-5 = \boxed{-7}$

use correct notation!

(b) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \dfrac{x^2-3x-10}{(x+2)} = \dfrac{9-9-10}{3+2} = \dfrac{-10}{5} = \boxed{-2}$

(c) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x+1 = 2(3)+1 = \boxed{7}$

9. (6 points) Calculate $\frac{dy}{dx}$ for the equation below:

$$\frac{d}{dx}\left((x^2+y^3)^5 = x^3\right)$$

$$\frac{5(x^2+y^3)^4 \cdot \left(2x+3y^2\frac{dy}{dx}\right)}{5(x^2+y^3)^4} = \frac{3x^2}{5(x^2+y^3)^4}$$

$$\cancel{2x} + 3y^2\frac{dy}{dx} = \frac{3x^2}{5(x^2+y^3)^4} - 2x$$
$$\cancel{-2x}$$

$$\frac{3y^2\frac{dy}{dx}}{3y^2} = \frac{\left(\frac{3x^2}{5(x^2+y^3)^4} - 2x\right)}{3y^2}$$

$$\boxed{\frac{dy}{dx} = \frac{\left(\frac{3x^2}{5(x^2+y^3)^4} - 2x\right)}{3y^2}}$$

Extra Credit(2 points) Bob tells you that if you have two functions multiplied together, to take the derivative you do the following: you just take the derivative of each function, then multiply the derivatives together. Is Bob right? If not, do an example to show him why he is wrong.

Bob is wrong. if $f(x) = (x+2)(x+3)$.

Bob says $f'(x) = 1 \cdot 1 = 1$, when he should

use the product rule to get

$f'(x) = 1(x+3) + 1\cdot(x+2)$
$= 2x+5$