Minitest 4A - MTH 1310

Dr. Graham-Squire, Fall 2014

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 2 questions of the test, however you should still show all of your work. No calculators are allowed on the last 3 questions.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
6. Make sure you sign the pledge above.
7. Number of questions = 5. Total Points = 25.

1. (5 points) The element Dominicium has a half-life of 14 years, and decays according to an exponential decay model. How long has something been decaying if there is only 11% left of the original specimen? Round to the nearest 0.1 years.

$$A = Pe^{rt}$$

$$\frac{\frac{1}{2}P}{P} = \frac{Pe^{r14}}{P}$$

$$\ln \frac{1}{2} = \cancel{\ln e}^{14r}$$

$$\frac{\ln \frac{1}{2}}{14} = \frac{14r}{14}$$

$$r = \frac{\ln 0.5}{14}$$

$$A = Pe^{\left(\frac{\ln 0.5}{14}\right)t}$$

11% means

$$\frac{0.11 \cancel{P}}{\cancel{P}} = \frac{\cancel{P}e^{\left(\frac{\ln 0.5}{14}\right)t}}{\cancel{P}}$$

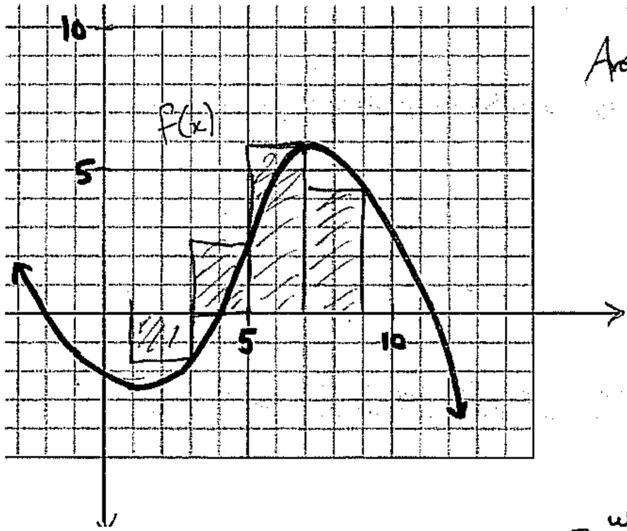
$$\ln 0.11 = \cancel{\ln e}^{\left(\frac{\ln 0.5}{14}\right)t}$$

$$\frac{\ln 0.11}{\left(\frac{\ln 0.5}{14}\right)} = \frac{\left(\frac{\ln 0.5}{14}\right)t}{\left(\frac{\ln 0.5}{14}\right)}$$

$$t = \frac{\ln 0.11}{\left(\frac{\ln 0.5}{14}\right)} \approx 44.6$$

$$\boxed{44.6 \text{ years}}$$

2. (5 points) Answer each of the questions in regards to the graph of $f(x)$ below:



$$\text{Area} \approx 2\cdot(-1.6) + 2(2.4) + 2(6) + 2(4.3)$$
$$= -3.2 + 4.8 + 12 + 8.6$$
$$= \boxed{22.2}$$

width $= \frac{8}{4} = 2$

heights of rectangles $\approx$
$-1.6,\ 2.4,\ 6,\ 4.3$

(a) Use a Riemann sum with 4 subintervals, evaluating at *right* endpoints, to estimate the value of the definite integral $\int_1^9 f(x)\,dx$. Estimate to the nearest 0.1.

(b) Is your answer from (a) an overestimate or an underestimate? Explain your reasoning.

(c) Is $\int_1^9 f(x)\,dx$ equal to the area between $f(x)$ and the x-axis? If not, explain how they are different.

(b) It looks like an overestimate. ~~Rectangles~~ The 3 leftmost rectangles are overestimates because they are above the graph, and the right rectangle is under the graph. On balance, it is an overestimate.

(c) Not equal. $\int_1^9 f(x)dx$ gives you the positive area above minus the area below, so it counts some area as positive and other area as negative (as opposed to calculating all area as positive, as we normally do).

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3. (5 points) Evaluate the integral:

$$\int x^2 \left(\frac{1}{x^6} - \frac{3}{x^3} + x^3 \right) dx$$

$$= \int \left(\frac{x^2}{x^6} - \frac{3x^2}{x^3} + x^5 \right) dx$$

$$= \int \left(x^{-4} - 3 \cdot \frac{1}{x} + x^5 \right) dx$$

$$= \boxed{\frac{1}{-3} x^{-3} - 3 \ln x + \frac{1}{6} x^6 + C}$$

$2x^{-1}$

4. (5 points) Evaluate the integral:

$$\int \frac{e^{2/x}}{x^2}\,dx$$

$u = \frac{2}{x}$

$du = -2x^{-2}\,dx$

$\frac{x^2}{-2}\,du = dx$

$$= \int \frac{e^u}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{-2}\,du$$

$$= \frac{-1}{2}\int e^u\,du$$

$$= \frac{-1}{2} e^u + C$$

$$= \boxed{\frac{-1}{2} e^{2/x} + C}$$

5. (5 points) Evaluate the integral:

$$= \frac{1}{x}$$

$$\int_e^4 (x + x^{-3} + x^{-1})\, dx$$

Simplify your answer as much as possible, but leave it in exact form (no decimal approximation, which you couldn't do anyway because this is the no calculator portion of the test).

$$= \frac{1}{2}x^2 + \left(\frac{1}{-2}x^{-2}\right) + \ln x \Big|_e^4$$

$$= \frac{1}{2}(4^2) + \frac{1}{-2\cdot 4^2} + \ln 4 - \left(\frac{1}{2}e^2 - \frac{1}{2}\cdot\frac{1}{e^2} + \ln e\right)$$

$$= 8 - \frac{1}{32} + \ln 4 - \frac{e^2}{2} + \frac{1}{2e^2} - 1$$

$$\cancel{\frac{255}{32}}$$

$$= \boxed{7 - \frac{1}{32} + \ln 4 - \frac{e^2}{2} + \frac{1}{2e^2}}$$

$$\frac{223}{32}$$

Extra Credit (1 point) Evaluate the integral $\int_{-1}^{2} \frac{1}{x^4}\, dx$, and sketch a graph to help confirm that your answer is correct.

$\int_{-1}^{2} \frac{1}{x^4}\, dx$ <u>DNE</u> b/c $\frac{1}{x^4}$ is <u>not</u> defined at $x=0$!

area makes no sense

b/c it has no top / is infinite!