

Test 3A - MTH 1310

Dr. Graham-Squire, Fall 2012

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are not allowed on this test. All cell phones should be turned off and put away, if I see a cell phone out it will be considered an honor code violation.
4. Calculators are allowed on the first 5 questions of the test, however you should still show all of your work. No calculators are allowed on the last 4 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need to use the quadratic formula, it is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 8. Total Points = 80.

1. (12 points) You are given that

$$f(x) = \frac{x}{1+x^2}, \quad f'(x) = \frac{1-x^2}{(1+x^2)^2}, \quad \text{and } f''(x) = \frac{-2x(3-x^2)}{(1+x^2)^3}$$

Answer the following questions, and make sure to show your work.

- (a) On what interval(s), if any, is f increasing?
- (b) On what interval(s), if any, is f concave down?
- (c) What are the (x, y) -coordinates of the local maximum(s), if any, of f ?
- (d) What are the (x, y) -coordinates of the inflection point(s), if any, of f ?

2. (10 points) Use calculus to find the absolute maximum and minimum of

$$f(x) = \frac{2}{3}x^3 - \frac{x^2}{2} - 3x + 2$$

on the interval $[-2,4]$.

3. (12 points) Eva wants to paint the inside of a rectangular room with a square base. The room must have a volume of 1200 ft^3 . The paint for the walls costs $\$0.05$ per ft^2 and the paint for the ceiling costs $\$0.02$ per ft^2 .
- (a) Find the dimensions of the room that will minimize the cost of the paint. Round your answers to the nearest 0.01 ft.
- (b) Use either the first derivative test or the second derivative test to confirm that your answer is a minimum.

4. (8 points) The percentage of families that were married households between 1970 and 2000 is approximately

$$P(t) = 86.9e^{-0.05t} \quad (0 \leq t \leq 3)$$

where t is measured in decades, with $t = 0$ corresponding to 1970.

- (a) What was the percentage of families that were married in 1980? Round to the nearest 0.01.
- (b) In what year (approximately, round to the nearest year) did the percent of married households drop to 80.6%?

5. (6 points) Adam bought a house in 2012 for \$292,000. Assuming that the worth of the house increases by 2.3% a year, compounded continuously, in what year will the house be worth \$350,000? Round to the nearest year.

NO CALCULATORS FOR THIS PART

6. (8 points) Use implicit differentiation to find $\frac{dy}{dx}$ for the equation

$$7x + \frac{x}{y} + y^4 = 13.$$

7. (10 points) You are backpacking in the woods when you make a terrible puppy-killing math mistake. It is so bad that a tree falls on your leg, and won't release you until you answer the following questions without the use of a calculator:

The relationship between Weasel Realty's yearly profits, $P(x)$, and the amount of money x spent on advertising per year is given by

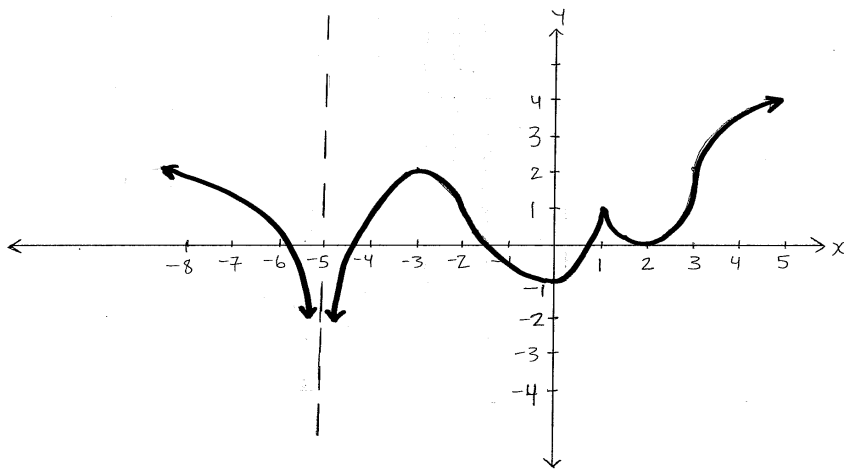
$$P(x) = \frac{-1}{8}x^2 + 7x + 30 \quad (0 \leq x \leq 50)$$

where both $P(x)$ and x are measured in thousands of dollars.

(a) *Use differentials* to estimate the change in profits when advertising expenditure is increased from \$24,000 to \$26,000.

(b) Is it a good idea to increase the advertising expenditure described above? Explain why or why not.

8. (8 points) Answer the questions for the graph of the function $f(x)$.



(a) On what interval(s), if any, is f decreasing?

(b) On what interval(s), if any, is f concave up?

(c) What are the (x, y) -coordinates of the local minimum(s), if any, of f ?

(d) What are the (x, y) -coordinates of the inflection point(s), if any, of f ?

9. (6 points) Use logarithm rules to expand and simplify, as much as possible, the expression

$$\ln \frac{x^2}{e^{\sqrt{x}}(1+x)^2}$$

Extra Credit(2 points maximum) Choose whether you want 1 point extra credit or 2 points extra credit. If you ask for 1, you are guaranteed to get it. If you ask for 2, and more than 4 other people in the class also ask for 2, then you all get zero for the extra credit.