

Business Calculus Test 3 Review Answers

Dr. Graham-Squire, Fall 2012

1. Find $\frac{dy}{dx}$ for the equation $x^3 + xy^2 + y^3 = 0$.

Ans: $\frac{dy}{dx} = \frac{-3x^2 - y^2}{2xy + 3y^2}$

2. Let f be the function defined by $y = f(x) = \frac{2x^2 + 1}{x + 1}$. Find the differential of f and use it to find the approximate change in y if x changes from 1 to 1.1.

Ans: $dy = \frac{2x^2 + 4x - 1}{(x + 1)^2} \cdot dx, dy = 0.125.$

3. For each function, find
- (a) the intervals where the function is increasing or decreasing,
 - (b) any relative maximum or minimum points (if any),
 - (c) the intervals where f is concave up or down, and
 - (d) inflection points (if any). For fun, you can also
 - (e) sketch a graph of the function from the information you found, then compare to what you get when you put it into a graphing calculator.

(i) $f(x) = x^4 - 2x^2$

(ii) $f(x) = x\sqrt{x-1}$

Ans: (i) is increasing on $(-1,0)$ and $(1,\infty)$, decreasing on $(-\infty,-1)$ and $(0,1)$. minimums at $(\pm 1, -1)$ and a max at $(0,0)$. It is concave up on $(-\infty, -1/\sqrt{3})$ and $(1/\sqrt{3}, \infty)$ and concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$, with inflection points at $(\pm 1/\sqrt{3}, -1/9)$.

(ii) is always increasing, has no max or min. Concave down on $(1, 8/3)$ and concave up on $(8/3, \infty)$, with inflection point at $(8/3, 8\sqrt{5}/3\sqrt{3})$.

4. Find the absolute maximum and minimum (if they exist) of the function $g(x) = x\sqrt{4-x^2}$ on the interval $[0,2]$.

Ans: Absolute min of 0 at both $x = 0$ and $x = 2$, abs. max of 2 at $x = \sqrt{2}$.

5. A rectangular box is to have a square base and a volume of 20 ft^3 . If the material for the base costs 30 cents/ ft^2 , the material for the top costs 20 cents/ ft^2 , and the material for the sides costs 20 cents/ ft^2 , determine the dimensions of the box that give a minimum cost. Check your answer to make sure it is a minimum.

Ans: 2.52 ft by 2.52 ft by 3.15 ft (where the 3.15 is the height). You also need to use the first or second derivative test to confirm that your answer is a minimum.

6. The number of internet users in China is approximated by the function

$$N(t) = 94.5e^{0.2t} \quad (1 \leq t \leq 6)$$

where $N(t)$ is measured in millions and t is years with $t = 1$ being 2005.

- (a) How many users are there in 2010?

Ans: 313,751,049 users.

- (b) When did the number of users equal 190,300,000?

Ans: When $t = 3.5$, so approximately the middle of 2007.

7. Expand and simplify the expression $\ln \frac{x^2 \cdot e^{3x}}{\sqrt{x}(1+x)^2}$.

Ans: $1.5 \ln x + 3x - 2 \ln(1+x)$

8. Find the interest rate needed for an investment of \$4000 to double in 5 years if interest is compounded continuously.

Ans: 13.86%