

Test 2^D - MTH 1310
Dr. Graham-Squire, Fall 2012

8:59

Name: Key

total: 19 min.

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 6 questions of the test, however you should still show all of your work. No calculators are allowed on the last 3 questions.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you need to use the quadratic formula, it is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
6. Make sure you sign the pledge.
7. Number of questions = 9. Total Points = 90.

1. (10 points) For a certain brand of TV, the revenue and cost functions are given by

$$R(x) = 500x - 0.04x^2$$

and

$$C(x) = 0.000002x^3 - 0.05x^2 + 400x + 90,000$$

where $R(x)$ and $C(x)$ are in dollars and x is the number of TVs produced.

(a) Find the profit function $P(x)$. ✓

(b) Find $P'(5000)$. What does it mean? ← Round to nearest whole number. ✓✓✓✓

✓✓✓ (c) Find $P'(6500)$. What does this tell you about what the appropriate level of production should be?

$$\begin{aligned} \text{(a) } P(x) &= R(x) - C(x) = 500x - 0.04x^2 - (0.000002x^3 - 0.05x^2 + 400x + 90,000) \\ &= -0.000002x^3 + 0.01x^2 + 100x - 90,000 \end{aligned}$$

$$\text{(b) } P'(x) = -0.000006x^2 + 0.02x + 100$$

$$P'(5000) \approx 50$$

Means that the 5001st TV makes about \$50 profit.

$$\text{(c) } P'(6500) \approx -24$$

The 6501st TV is actually losing money because $P'(6500)$ is negative, so the optimal production must be something

less than 6500, but greater than 5000.

— | if don't state optimal production.

2. (12 points) Let $f(x) = \begin{cases} (x-2) & \text{if } x \leq 3 \\ \frac{x^2-4}{x^2-5} & \text{if } 3 < x < 4 \\ x+7 & \text{if } x \geq 4 \end{cases}$

(a) Find the following limits: Write DNE if limit does not exist



\checkmark (i) $\lim_{x \rightarrow 3^-} f(x) = \frac{1}{5} = \frac{3-2}{9-4}$ \checkmark (ii) $\lim_{x \rightarrow 3^+} f(x) = 4$ \checkmark (iii) $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

\checkmark (iv) $\lim_{x \rightarrow 4^-} f(x) = 11$ \checkmark (ii) $\lim_{x \rightarrow 4^+} f(x) = 11$ \checkmark (iii) $\lim_{x \rightarrow 4} f(x) = 11$

$4^2 - 5 = 16 - 5$ $4 + 7 = 11$

(b) Find all values of x where the function $f(x)$ is not continuous. Give a brief explanation/reason why the function is not continuous at that value of x . (Hint: for a function to be discontinuous it must fail one part of the definition of continuity.)

$f(x)$ is not continuous at $x=3$ because the limit does not exist. $\checkmark \checkmark \checkmark$

$\frac{x-2}{(x-2)(x+2)}$ shows that the function is not defined \checkmark at $x=2$ or $x=-2$, so the function is discontinuous \checkmark at those points also.

$x=3, x=2, \text{ and } x=-2$

3. (12 points) Calculate the limits. Make sure you use correct notation or you will lose points.

If limit dne, write does not exist

(a) $\lim_{x \rightarrow 2} \frac{3x}{x^2 - 9} = \frac{6}{2^2 - 9} = \boxed{\frac{6}{-5}}$

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(b) $\lim_{x \rightarrow (-5)} \frac{x^2 + 10x + 25}{x^2 - 25} = \lim_{x \rightarrow (-5)} \frac{(x+5)(x+5)}{(x-5)(x+5)} = \frac{0}{-10} = \boxed{0}$

-1 for bad notation

5

(c) $\lim_{x \rightarrow (-\infty)} \frac{3x^6 + 7x^4 - 100}{8x^2 + 11} = \lim_{x \rightarrow (-\infty)} \frac{3x^4 + 7x^2 - \frac{100}{x^2}}{8 + \frac{11}{x^2}}$

$= \frac{\infty + \infty - 0}{8 + 0} = \infty$

-1 for bad notation

4

limit does not exist.

\boxed{DNE}

10:24
10:32

8

4. (10 points) For the hit song "Mo math no problems", the total worldwide revenues are approximated by the function

$$M(x) = \frac{140x^2}{x^2 + 2}$$

where $M(x)$ is measured in millions and x is the number of years since the songs release.

(a) How fast are the total revenues changing 3 years after the songs release? Round to the nearest 0.01.

(b) What will be the total revenues in the long run?

$$(a) \quad M'(x) = \frac{280x(x^2+2) - 140x^2(2x)}{(x^2+2)^2} = \frac{280x^3 + 560x - 280x^3}{(x^2+2)^2}$$

$$= \frac{560x}{(x^2+2)^2}$$

$M'(3) =$ ~~88.84~~ million dollars/year ✓
 " $\frac{560 \cdot 3}{11^2} = 13.88$ ✓

(b) $\lim_{x \rightarrow \infty} \frac{140x^2 \cdot \frac{1}{x^2}}{x^2+2 \cdot \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{140}{1 + \frac{2}{x^2}} = 140$ million dollars! ✓

5. (10 points) Steve leads backpacking trips for groups of people. He needs a group of at least 20 to run a trip, and if there are only 20 people then he charges them \$200 per person. If there are more than 20 people in the group (up to a maximum of 35), he charges \$4 less per person (thus if there are 21 people in the group, each person in the group only has to pay \$196). Let x be the number of people in the group above 20. Write an equation that gives the total revenue Steve will get from a trip, in terms of x .

$$\text{Revenue} = (\# \text{ of people on trip}) \cdot (\text{cost per person}) \quad \checkmark \checkmark \checkmark$$

$$x = \# \text{ of people } \underline{\text{over}} \ 20$$

$$\Rightarrow \text{total } \# \text{ of people} = 20 + x \quad \checkmark \checkmark \checkmark$$

cost goes down \$4 per person over 20, so for

x people over 20 it goes down $4x$.

$$\Rightarrow \text{cost per person} = 200 - 4x \quad \checkmark \checkmark \checkmark$$

$$R(x) = (20 + x)(200 - 4x) \quad \checkmark$$

NO CALCULATOR

Name: Key

6. (10 points) Find the derivative of

$$f(x) = ((x-3)(x^4-3x))^8.$$

You do not need to simplify your answer.

$$f'(x) = 8 \left((x-3)(x^4-3x) \right)^7 \cdot \left(1 \cdot (x^4-3x) + (4x^3-3)(x-3) \right)$$

$$x^4 - 3x + 4x^4 - 3x - 12x^3 + 9$$

$$5x^4 - 12x^3 - 6x + 9$$

-1 for notation problems.

7. (8 points) Find the second derivative $f''(x)$ of the function

$$f(x) = (x^3 - 9)^6.$$

-1 or 2 for bad notation

$$\checkmark f'(x) = 6(x^3 - 9)^5 (3x^2)$$

$$f'(x) = 18x^2(x^3 - 9)^5 \quad \checkmark \checkmark$$

$$f''(x) = 18(2x(x^3 - 9)^5 + x^2 \cdot 5(x^3 - 9)^4(3x^2)) \quad \checkmark \checkmark \checkmark$$

$$= 18(2x(x^3 - 9)^5 + 15x^4(x^3 - 9)^4)$$

$$= 36x(x^3 - 9)^5 + 270x^4(x^3 - 9)^4$$

$$\checkmark = 18x(x^3 - 9)^4(2(x^3 - 9) + 15x^3)$$

$$f''(x) = 18x(x^3 - 9)^4(17x^3 - 18)$$

8. (8 points) Find $f'(x)$ if $f(x) = \frac{x^7 - 3x^3 - \sqrt[3]{x}}{x^4}$. Simplify your answer.

$$f(x) = \frac{x^7}{x^4} - \frac{3x^3}{x^4} - \frac{x^{1/3}}{x^4} \quad \checkmark\checkmark\checkmark$$

$$\frac{1}{3} - 4 = \frac{1}{3} - \frac{12}{3} = -\frac{11}{3}$$

$$f(x) = x^3 - 3x^{-1} - x^{-11/3}$$

$$f'(x) = 3x^2 + 3x^{-2} + \frac{11}{3}x^{-14/3} \quad \checkmark\checkmark\checkmark$$

or

$$f'(x) = \frac{x^4(7x^6 - 9x^2 - \frac{1}{3}x^{-4/3}) - 4x^3(x^7 - 3x^3 - x^{1/3})}{x^8}$$

$$= \frac{7x^{10} - 9x^6 - \frac{1}{3}x^{10/3} - 4x^{10} + 12x^6 + 4x^{4/3}}{x^8}$$

$$= \frac{3x^{10} + 3x^6 + \frac{11}{3}x^{10/3}}{x^8}$$

⊙

9. (10 points) (a) Find $f'(x)$ if $f(x) = x^2(3x^3 + 2x - 10)$. Simplify your answer.

$$f(x) = 3x^5 + 2x^3 - 10x^2 \quad \checkmark\checkmark$$

$$f'(x) = 15x^4 + 6x^2 - 20x \quad \checkmark\checkmark\checkmark$$

$$x(15x^3 + 6x - 20)$$

(b) Find the equation of the tangent line at $x = 1$.

$$f(1) = 1(3 + 2 - 10) = -5 \quad \checkmark$$

$$f'(1) = 15 \cdot 1 + 6 \cdot 1 - 20 = 1 \quad \checkmark\checkmark$$

$$y - (-5) = 1(x - 1) \quad \checkmark$$

$$y = x - 1 - 5$$

$$y = x - 6 \quad \checkmark$$

Extra Credit(2 points) A cannon shoots a cannonball straight up into the air, and the height of the cannonball is given by the function $h(t)$. You are told that $h'(t) = -4t^3 - 34t^2 - 6$. Does this seem to be accurate? Why or why not?

Not accurate b/c $h'(t)$ is always negative,
and velocity should be positive b/c cannonball
is going up.