Quiz 5A, Business Calculus

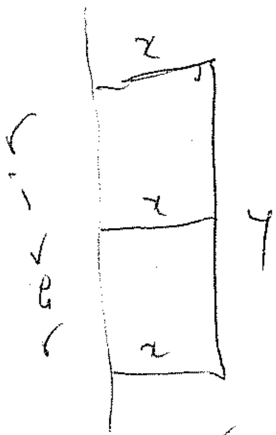
Fall 2012

Name: Key

1. (4 points) Bob wants to build a rectangular holding pen for his cows and goats. The pen will be placed next to a river so that the animals can drink, so he only needs fencing for 3 sides. He also needs to put in fencing down the middle of the pen (perpendicular to the river) so that the cows and goats are separated. Thus there will be one side of fence parallel to the river, and 3 sides perpendicular. The total size of the pen has to be 500 ft^2.

   (a) Draw a diagram of the situation. Include any variables you will use in your solution.

   (b) Use calculus to find the minimum total amount of fencing Bob will need to build the pen. Round your answer to the nearest whole number. You do not need to prove that your answer is a minimum.



$3x + y =$ total fencing

$500 = xy$

$y = \frac{500}{x}$

$\Rightarrow 3x + \frac{500}{x} = f(x)$

$3 - \frac{500}{x^2} = f'(x)$

$3 - \frac{500}{x^2} = 0$

$\sqrt{x^2} = \sqrt{\frac{500}{3}}$

$x = \sqrt{\frac{500}{3}} = 12.9099$

$f(12.91) = 77.45$

$\approx \boxed{77 \text{ feet}}$

2. (4 points) Let $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 \overset{-}{\cancel{+}} x^3$. Find the absolute maximum and absolute minimum for f on the interval [-3,2]. Round to nearest 0.1.

$f'(x) = x^4 - 2x^3 - 3x^2 = x^2(x^2 - 2x - 3) = x^2(x-3)(x+1)$ ✓

$f'(x) = 0$ at $x = 0, 3, -1$ ✓

$f(0) = 0$ ✓

$\boxed{\cancel{f(3)} = -18.9}$ Not in interval ✓

$f(-1) = \boxed{0.3} \leftarrow$ Max ✓

$f(-3) = \frac{-241}{5} - \frac{81}{2} + 27 = \boxed{-61.7} \leftarrow$ Min ✓

$f(2) = \frac{32}{5} - 8 - 8 = -9.6$ ✓

3. (2 points) Solve for x: $\frac{2^{5x-4}}{8} = 2^{4x+3}$.

$$\frac{2^{5x-4}}{2^3} = 2^{4x+3}$$

$$2^{5x-4-3} = 2^{4x+3}$$

$$\Rightarrow 5x - 7 = 4x + 3$$

$$\boxed{x = 10}$$

Quiz 5B, Business Calculus

Fall 2012

Name: Key

1. (4 points) Let $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 \cancel{+} - x^3$. Find the absolute maximum and absolute minimum for f on the interval [-0.5,4]. Round to nearest 0.1.

$$f'(x) = x^4 - 2x^3 \cancel{+} - 3x^2 = x^2(x^2 - 2x - 3) = x^2(x-3)(x+1)$$

$$f'(x) = 0 \quad \text{at} \quad x = 0, 3, -1$$

$$f(0) = 0$$

$$f(3) = \boxed{-18.9 \longleftarrow \text{Min}}$$

$$\cancel{f(-1) \approx 0.3} \quad \text{Not in interval}$$

$$f(-0.5) = 0.0875$$

$$f(4) \approx \boxed{12.8 \longleftarrow \text{Max}}$$

2. (2 points) Solve for x: $\frac{3^{8x-3}}{9} = 3^{7x}$.

$$\Rightarrow \frac{3^{8x-3}}{3^2} = 3^{7x}$$

$$3^{8x-3-2} = 3^{7x}$$

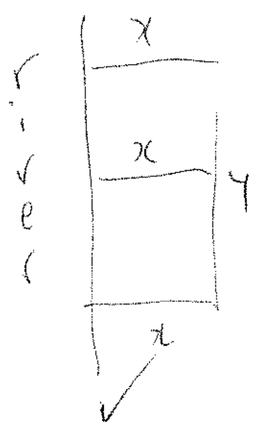
$$\Rightarrow 8x - 5 = 7x$$

$$\boxed{x = 5}$$

3. (4 points) Bob wants to build a rectangular holding pen for his cows and goats. The pen will be placed next to a river so that the animals can drink, so he only needs fencing for 3 sides. He also needs to put in fencing down the middle of the pen (perpendicular to the river) so that the cows and goats are separated. Thus there will be one side of fence parallel to the river, and 3 sides perpendicular. The total size of the pen has to be 700 ft^2.

(a) Draw a diagram of the situation. Include any variables you will use in your solution.

(b) Use calculus to find the minimum total amount of fencing Bob will need to build the pen. Round your answer to the nearest whole number. You do <u>not</u> need to prove that your answer is a minimum.



$3x + y = \text{total fencing}$

$xy = 700$

$y = \frac{700}{x}$

$3x + \frac{700}{x} = f(x)$

$3 - \frac{700}{x^2} = f'(x)$

$3 - \frac{700}{x^2} = 0$

$x^2 = \frac{700}{3}$

$x = \sqrt{\frac{700}{3}} = 15.275$

$f(15.275) = 91.6$

$= \boxed{92 \text{ feet}}$

Quiz 5C, Business Calculus

Fall 2012

9:24

9:32

8

⇒ 20 minutes. Maybe more?

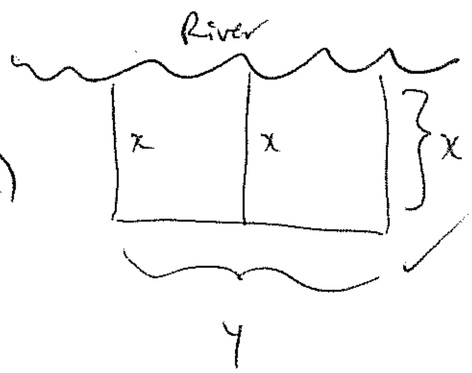
Name: Key

1. (4 points) Bob wants to build a rectangular holding pen for his cows and goats. The pen will be placed next to a river so that the animals can drink, so he only needs fencing for 3 sides. He also needs to put in fencing down the middle of the pen (perpendicular to the river) so that the cows and goats are separated. Thus there will be one side of fence parallel to the river, and 3 sides perpendicular. The total size of the pen has to be 400 ft^2.

   (a) Draw a diagram of the situation. Include any variables you will use in your solution.

   (b) Use calculus to find the minimum total amount of fencing Bob will need to build the pen. Round your answer to the nearest whole number. You do <u>not</u> need to prove that your answer is a minimum.

(a)



(b) $400 = xy \implies y = \frac{400}{x}$

total fencing $= 3x + y$

$$f(x) = 3x + \frac{400}{x}$$

$$f'(x) = 3 - \frac{400}{x^2}$$

$$0 = 3 - \frac{400}{x^2}$$

$$x^2 = \frac{400}{3}$$

$$x = \sqrt{\frac{400}{3}} \approx 11.54$$

$$f(11.54) \approx 69.28$$

$\Rightarrow$ total fencing $= \boxed{69 \text{ feet}}$

2. (2 points) Solve for x: $\dfrac{4^{2x+3}}{16} = 4^{3x-5}$.

$$\frac{4^{2x+3}}{4^2} = 4^{3x-5}$$

$$4^{2x+3-2} = 4^{3x-5}$$

$$\Rightarrow 2x+1 = 3x-5$$

$$\boxed{6 = x}$$

3. (4 points) Let $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 - x^3$. Find the absolute maximum and absolute minimum for f on the interval $[-2,4]$.

$$f'(x) = x^4 - 2x^3 - 3x^2 = x^2(x^2-2x-3) = x^2(x+1)(x-3)$$

$$\Rightarrow f'(x) = 0 \quad \text{at} \quad x = 0, -1, 3$$

check $f(-1) = .3$

$f(0) = 0$

$f(3) = \boxed{-18.9 \leftarrow \text{Min}}$

$f(4) = \boxed{12.8 \leftarrow \text{Max}}$

$f(-2) = -6.4$