# Quiz 6, Abstract Algebra

Dr. Graham-Squire, Spring 2016

**Name:** _____

1. (4 points) Let $G$ be an Abelian group and $|G| = 100$.

   (a) What are all possible $\mathbb{Z}_n$ groups (or direct products of $\mathbb{Z}_n$ groups) that $G$ could be isomorphic to?

   (b) If you knew $G$ was not cyclic and had exactly 2 elements of order 4, would that tell you which group $G$ is isomorphic to? If not, which groups could you eliminate? Justify your answer.

2. (3 points) The mapping $\phi : \mathbb{Z}_{20} \to \mathbb{Z}_8$ given by $\phi(x) = 2x$ is a homomorphism.

   (a) Is $\phi$ an onto mapping (that is, are all 8 elements of $\mathbb{Z}_8$ in the image of $\phi$)?

   (b) Calculate the order of the kernel (that is, find $|\text{Ker}\phi|$).

   (c) If all you knew was $\alpha : G \to H$ was a homomorphism, $|G| = 20$, and $|H| = 8$, could you tell whether or not $\alpha$ was onto? Explain why or why not.

3. (3 points) Let $G$ be the group of all polynomial functions with real coefficients, with *addition* as the group operation.

   (a) Prove that the derivative mapping $d : G \to G$ given by $d(f) = f'$ is a homomorphism.

   (b) What is the kernel of $d$?