

Quiz 5, Abstract Algebra

Dr. Graham-Squire, Spring 2016

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Name: _____

Key

1. (3 points) Let H be a subgroup of G , and suppose that H is also a subgroup of $Z(G)$ (where $Z(G)$ is the center of G , that is all $g \in G$ that commute with *everything* in G). Prove that H is normal in G .

H subgroup of $Z(G) \Rightarrow hx = xh$ for all $x \in G$ and $h \in H$.

Then the coset $xH = \{xh \mid h \in H\} = \{hx \mid h \in H\} = Hx$

b/c H is subgroup
of the center

Thus for all $x \in G$, $xH = Hx$

$\Rightarrow H \triangleleft G$.

1.5 for det. of Normal (or N.S.T.)

2. (4 points) (a) Describe the elements of the factor group $\mathbb{Z}_4 \oplus \mathbb{Z}_4 / \langle (1, 2) \rangle$.

(b) $\mathbb{Z}_4 \oplus \mathbb{Z}_4 / \langle (1, 2) \rangle$ is isomorphic to either \mathbb{Z}_4 or $\mathbb{Z}_2 \oplus \mathbb{Z}_2$. Which one is it? Explain your reasoning.

$$(a) \langle (1, 2) \rangle = \{ (1, 2), (2, 0), (3, 2), (0, 0) \} = H \quad \checkmark$$

So elements of $\mathbb{Z}_4 \oplus \mathbb{Z}_4 / \langle (1, 2) \rangle$ are

H

$$(0, 1) + H = \{ (1, 3), (2, 1), (3, 3), (0, 1) \}$$

$$(0, 2) + H = \{ (1, 0), (2, 2), (3, 0), (0, 2) \} \quad \checkmark$$

$$(0, 3) + H = \{ (1, 1), (2, 3), (3, 1), (0, 3) \}$$

$$(b) \text{ So } \langle (0, 1) + H \rangle = \{ (0, 1) + H, (0, 2) + H, (0, 3) + H, H \} \quad \checkmark$$

generates the factor group $\Rightarrow \approx$

$$\boxed{\mathbb{Z}_4}$$

\checkmark

3. (3 points) Recall that \mathbb{R}^* is the group with elements $\mathbb{R} - \{0\}$ and binary operation multiplication. Also, let \mathbb{R}^+ be the group of positive real numbers.

Prove that $\mathbb{R}^* = \mathbb{R}^+ \times \{-1, 1\}$.

(Recall that $G = H \times K \Leftrightarrow H, K$ are normal in G , $H \cap K = \{e\}$ and $G = HK$)

You can assume \mathbb{R}^+ and $\{-1, 1\}$

~~Since \mathbb{R}^* is Abelian, all subgroups are normal
 $\Rightarrow \mathbb{R}^+, \{-1, 1\} \leq \mathbb{R}^*$ are normal.~~

$\mathbb{R}^* \cap \{-1, 1\} = \{1\} = \{e\}$ b/c $-1 \notin \mathbb{R}^+$ ✓
 and $1 \in \mathbb{R}^+$ b/c it is positive.

Let $x \in \mathbb{R}^*$. If $x > 0$, $x = x \cdot 1$ ✓
 \uparrow \uparrow
 $\in \mathbb{R}^+$ $\in \{-1, 1\}$

If $x < 0$, $x = |x| \cdot (-1)$ ✓
 \uparrow \uparrow
 $\in \mathbb{R}^+$ $\in \{-1, 1\}$

So $\mathbb{R}^* = (\mathbb{R}^+) (\{-1, 1\})$

$\Rightarrow \mathbb{R}^* = \mathbb{R}^+ \times \{-1, 1\}$ □

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