Quiz 5, Abstract Algebra

Dr. Graham-Squire, Spring 2016

Name: ______________________________

1. (3 points) Let H be a subgroup of G, and suppose that H is also a subgroup of $Z(G)$ (where $Z(G)$ is the center of G, that is all $g \in G$ that commute with *everything* in G). Prove that H is normal in G.

2. (4 points) (a) Describe the elements of the factor group $\mathbb{Z}_4 \oplus \mathbb{Z}_4/\langle(1,2)\rangle$.

   (b) $\mathbb{Z}_4 \oplus \mathbb{Z}_4/\langle(1,2)\rangle$ is isomorphic to either $\mathbb{Z}_4$ or $\mathbb{Z}_2 \oplus \mathbb{Z}_2$. Which one is it? Explain your reasoning.

3. (3 points) Recall that $\mathbb{R}^*$ is the group with elements $\mathbb{R} - \{0\}$ and binary operation multiplication. Also, let $\mathbb{R}^+$ be the group of positive real numbers.

   Prove that $\mathbb{R}^* = \mathbb{R}^+ \times \{-1, 1\}$.

   (Recall that $G = H \times K \Leftrightarrow H \cap K = \{e\}$ and $G = HK$. You can assume $\mathbb{R}^+$ and $\{-1, 1\}$ are normal in $\mathbb{R}^*$)