Quiz 4, Abstract Algebra

Dr. Graham-Squire, Spring 2016

Name: Key

12:01
12:06
5 min.

1. (3 points) Suppose G is a group such $|G| = 200$, and K is a subgroup of G with $|K| = 20$. Suppose K is also a proper subgroup of H, and H is a proper subgroup of G. What is/are the possible order(s) of H? Explain your reasoning.

$|H|$ could be either 40 or 100. By Lagrange's Theorem, the order of a subgroup must divide the order of the larger group. Thus $|H|$ must divide 200 but be divisible by 20. The only such options are $\boxed{40}$ and $\boxed{100}$.

2. (4 points) Let H be a subgroup of G. Using your knowledge of cosets, explain in words what the following equation means and why it is true. You do not have to give a full proof, just an explanation will suffice:

   For any $a, b \in G$, $|aH| = |bH|$.

$|aH| = |bH|$ means that all cosets have the same number of elements. This makes sense because

Suppose $H = \{h_1, h_2, \dots, h_n\}$. Then $aH = \{ah_1, ah_2, \dots, ah_n\}$

so $|aH| = n$, and $bH = \{bh_1, bh_2, \dots, bh_n\} \Rightarrow |bH| = n$.

If H is infinite, then $|aH| = |bH| = \infty$.

3. (3 points) What is/are the possible order(s) of the nonidentity elements of $\mathbb{Z}_9 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_9$? Explain your reasoning.

An element of $\mathbb{Z}_9$ must have order that divides 9, so it could have order 1, 3 or 9. Then an element $(a,b,c) \in \mathbb{Z}_9 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_9$ would have $|(a,b,c)| = \text{lcm}(|a|, |b|, |c|)$. Since any orders for a, b, c would divide 9, could have at most $|(a,b,c)| = 9$. Could also have an order of 3 (e.g. $|(3,0,0)|$ or 1 $(|(0,0,0)|)$ and that is all. $\boxed{9, 3 \text{ or } 1}$