

Quiz 3, Abstract Algebra

Dr. Graham-Squire, Spring 2016

Name: _____

1. (5 points) Prove that \mathbb{Z}_n is isomorphic to *any* cyclic group of order n . Hint: use the fact that the cyclic group can be expressed as $\langle a \rangle$ for some a in the group, an arbitrary element in the group looks like a^j .

2. (2 points) Give at least one reason why \mathbb{Z} (the integers) under addition is NOT isomorphic to \mathbb{Q} (the rationals) under addition. You don't need a full proof or justification, just give a reason. (In other words, this should be a short answer).

3. (3 points) Are the following two groups isomorphic or not? You do not have to give a full proof, but if you think they are isomorphic you should give a good explanation of why you think so. If you think they are NOT isomorphic, you should explain that also. Your explanation can either come from your understanding of the groups themselves, or from the Cayley tables below (preferably both), but it should be justified by something.

The groups are:

- (i) $\{(1), (123456), (135)(246), (14)(25)(36), (153)(264), (165432)\}$ (a subgroup of S_6) and
 (ii) $\{R_0, R_{120}, R_{240}, F_{30}, H, F_{150}\}$ (a subgroup of D_6 . H is the horizontal flip, and F_{30} and F_{150} are the two flips that are 30 degrees off of vertical).

The two groups have the following Cayley tables, with elements in the permutation group listed as e, a, b, \dots, f (respectively, with e as the identity) and elements in the subgroup of D_6 listed as $1, 2, 3, \dots, 6$ (respectively). Note: There are many ways to make a mapping between the two sets—you should NOT assume that, for example, element b must map to 3 just because they are both the third things on each list.

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	c	d	f	e
b	b	c	d	f	e	a
c	c	d	f	e	a	b
d	d	f	e	a	b	c
f	f	e	a	b	c	d

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	1	5	6	4
3	3	1	2	6	4	5
4	4	6	5	1	3	2
5	5	4	6	2	1	3
6	6	5	4	3	2	1