Quiz 1, Abstract Algebra

Dr. Graham-Squire, Spring 2016

7:53
8:00
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⇒ give 20 min?

Name: Key

6 points

1. Consider the following 3 sets. For each set, determine if it is a group or not. If the set is a group, prove that it satisfies the conditions to be a group. If the set is *not* a group, explain what part of the definition of *group* that it fails.

   (a) The odd integers, under addition

   (b) $\mathbb{Z}_{10}$, under addition mod 10

   (c) The irrational numbers, under multiplication

✓ (a) Not a group → has no identity (not zero)
→ not closed ($1+1=2$ ← Not odd)

(b) Is a group!

✓ Closed: Adding two elements gives another integer, and when you do mod 10, always get an integer between 0 and 9.

✓ Associative: $\mathbb{Z}_{10}$ inherits associativity from $\mathbb{Z}$, since the addition is the same.

✓ Identity: $0 \in \mathbb{Z}_{10}$ is the identity

✓ Inverses: 0 is its own identity, and for any other $a \in \mathbb{Z}_{10}$, have $10-a = a^{-1}$ b/c
$(10-a)+(a) = 10 = 0 \mod 10$ ∎

✓ (c) Not a group → has no identity ($1 \notin \mathbb{I}$)
→ not closed! $\sqrt{2} \cdot \sqrt{2} = 2$ ← Not irrational!

4 points

2. Let G be an Abelian group (that means that $ab = ba$ for all $a, b \in G$). Let H be the subset of G given by

$$H = \{g^2 \,|\, g \in G\}$$

In other words, H is the subset of "squares" of elements of G. Use either the one-step or the two-step subgroup test to prove that H is a subgroup of G.

<u>Two step:</u> Let $a, b \in H$. Then $a = x^2$, $b = y^2$ for some $x, y \in G$. Then $xy \in G$ (b/c G is a group)

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and $(xy)^2 = (xy)(xy) = x^2y^2 = ab \in H$.

b/c G is Abelian

Let $a \in H$. Then $a = x^2$ for some $x \in G$. Since $x \in G$, $x^{-1} \in G$ also,

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$\Rightarrow (x^{-1})^2 = x^{-2} \in H$

and $a(x^{-2}) = x^2x^{-2} = e$

$\Rightarrow x^{-2} = a^{-1} \in H$